Inverse Trigonometric Functions

$\sin^{-1}\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ $\cos^{-1}[0, \pi]$ $\tan^{-1}\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ $\cot^{-1}(0, \pi)$ $\pi \approx 3.14$ for locating sin 3, cos 10, tan 6

1 Why Inverse Trig Needs a Principal Value Branch

$\sin^{-1} : [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\cos^{-1} : [-1, 1] \to [0, \pi]$

$\tan^{-1} : \mathbb{R} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\cot^{-1} : \mathbb{R} \to (0, \pi)$

$\sec^{-1} : |x| \geq 1 \to [0, \pi] - \left\{\frac{\pi}{2}\right\}$

$\operatorname{cosec}^{-1} : |x| \geq 1 \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

2 Graphs of Inverse Trig Functions and Principal Values

3 Domain, Range and Inequalities of Inverse Trig Expressions

4 Extreme-Value and Greatest Integer Equations in Inverse Trig

5 $\sin^{-1}(\sin x)$ and $\cos^{-1}(\cos x)$: Zig-Zag and Tent Graphs

$\sin^{-1}(\sin x) = x,\ |x| \leq \frac{\pi}{2}$

$\sin^{-1}(\sin x) = \pi - x,\ \frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$

$\sin^{-1}(\sin x) = x - 2\pi,\ \frac{3\pi}{2} \leq x \leq \frac{5\pi}{2}$

$\sin^{-1}(\sin x) = -\pi - x,\ -\frac{3\pi}{2} \leq x \leq -\frac{\pi}{2}$

$\cos^{-1}(\cos x) = x,\ 0 \leq x \leq \pi$

$\cos^{-1}(\cos x) = 2\pi - x,\ \pi \leq x \leq 2\pi$

$\cos^{-1}(\cos x) = x - 2\pi,\ 2\pi \leq x \leq 3\pi$

6 $\tan^{-1}(\tan x)$, $\cot^{-1}(\cot x)$, $\sec^{-1}(\sec x)$ and Solving with Graphs

7 The Reference Triangle: Converting Inverse Trig Functions

$\sin^{-1} x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}},\ 0 < x < 1$

$\tan^{-1} x = \sin^{-1}\frac{x}{\sqrt{1+x^2}} = \cos^{-1}\frac{1}{\sqrt{1+x^2}} = \cot^{-1}\frac{1}{x},\ x > 0$

8 Simplifying Inverse Trig Expressions by Substitution

9 Inverse Trig of −x and 1/x

$\sin^{-1}(-x) = -\sin^{-1} x$

$\tan^{-1}(-x) = -\tan^{-1} x$

$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$

$\cos^{-1}(-x) = \pi - \cos^{-1} x$

$\sec^{-1}(-x) = \pi - \sec^{-1} x$

$\cot^{-1}(-x) = \pi - \cot^{-1} x$

$\tan^{-1}\frac{1}{x} = \cot^{-1} x,\ x > 0$

$\tan^{-1}\frac{1}{x} = -\pi + \cot^{-1} x,\ x < 0$

$\tan^{-1} x + \tan^{-1}\frac{1}{x} = \pm\frac{\pi}{2}\ (x \gtrless 0)$

10 $\sin^{-1}x + \cos^{-1}x = \pi/2$ and the Complementary Identities

$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2},\ |x| \leq 1$

$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2},\ x \in \mathbb{R}$

$\sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2},\ |x| \geq 1$

11 Range and Extreme Values Using $\sin^{-1}x + \cos^{-1}x = \pi/2$

12 $\tan^{-1}x + \tan^{-1}y$: The Addition Formula and Its Cases

$\tan^{-1} x + \tan^{-1} y = \tan^{-1}\frac{x+y}{1-xy},\ xy < 1$

$= \pi + \tan^{-1}\frac{x+y}{1-xy},\ x, y > 0,\ xy > 1$

$= -\pi + \tan^{-1}\frac{x+y}{1-xy},\ x, y < 0,\ xy > 1$

$\tan^{-1} x - \tan^{-1} y = \tan^{-1}\frac{x-y}{1+xy},\ xy > -1$

$= \pi + \tan^{-1}\frac{x-y}{1+xy},\ x > 0 > y,\ xy < -1$

$= -\pi + \tan^{-1}\frac{x-y}{1+xy},\ x < 0 < y,\ xy < -1$

13 $2\tan^{-1}x$, $3\tan^{-1}x$ and $\tan^{-1}$ Equations

$2\tan^{-1} x = \tan^{-1}\frac{2x}{1-x^2}\ (|x| < 1);\ \pm\pi + \ldots\ (x \gtrless \pm 1)$

$3\tan^{-1} x = \tan^{-1}\frac{3x-x^3}{1-3x^2}\ \left(|x| < \frac{1}{\sqrt{3}}\right);\ \pm\pi + \ldots$

14 Telescoping $\tan^{-1}$ Series

$\tan^{-1}\frac{a-b}{1+ab} = \tan^{-1} a - \tan^{-1} b,\ ab > -1$

15 Sum and Difference of $\sin^{-1}$ and $\cos^{-1}$

$\sin^{-1} x + \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right),\ x^2 + y^2 \leq 1$

$= \pi - \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right),\ x^2 + y^2 > 1$

$\cos^{-1} x + \cos^{-1} y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$

$\cos^{-1} x - \cos^{-1} y = \pm\cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right)\ (x \leq y\ /\ x > y)$

16 $2\sin^{-1}x$, $3\sin^{-1}x$, $2\cos^{-1}x$, $2\tan^{-1}x$: Where Each Case Comes From

$2\sin^{-1} x = \sin^{-1}(2x\sqrt{1-x^2})\ \left(|x| \leq \frac{1}{\sqrt{2}}\right);\ \pm\pi - \ldots$

$3\sin^{-1} x = \sin^{-1}(3x - 4x^3)\ \left(|x| \leq \frac{1}{2}\right);\ \pm\pi - \ldots$

$2\cos^{-1} x = \cos^{-1}(2x^2 - 1)\ (x \geq 0);\ 2\pi - \cos^{-1}(2x^2 - 1)\ (x < 0)$

$2\tan^{-1} x = \sin^{-1}\frac{2x}{1+x^2}\ (|x| \leq 1);\ \pm\pi - \ldots$

$2\tan^{-1} x = \pm\cos^{-1}\frac{1-x^2}{1+x^2}\ (x \geq 0\ /\ x < 0)$

17 Multiple-Angle Formulas: Evaluating and Solving