MATHEMATICS · TRIGONOMETRY · REVISION SHEET

# Inverse Trigonometric Functions

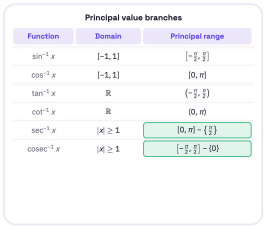
Inverse trigonometric functions run on two ideas: every function lives on its principal branch, and every formula has cases that come from where an angle can land. The points below are what JEE questions keep testing.

## If you only have five minutes

1. Principal ranges: $\sin^{-1}$, $\tan^{-1}$, $\operatorname{cosec}^{-1}$ around 0; $\cos^{-1}$, $\cot^{-1}$, $\sec^{-1}$ in $[0, \pi]$.
2. $f^{-1}(f(x)) = x$ only on the principal branch: zig-zag for $\sin^{-1}(\sin x)$, tent for $\cos^{-1}(\cos x)$.
3. $\sin^{-1} x + \cos^{-1} x = \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$: two unknowns become one.
4. $\tan^{-1} x + \tan^{-1} y$: check $xy$ against 1; $\sin^{-1} x + \sin^{-1} y$: check $x^2 + y^2$ against 1.
5. Multiple angles: substitute, read $\sin^{-1}(\sin \alpha)$ or $\cos^{-1}(\cos \alpha)$, translate the cut points back to $x$.

## 1 Why Inverse Trig Needs a Principal Value Branch

- $f^{-1}$ exists only for a bijection; its domain and range are the range and domain of $f$.
- Trig functions are many-one, so each is cut to a principal value branch first.
- $\sin^{-1} x$ is the angle in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ whose sine is $x$ (arcsin $x$).




### STANDARD RESULTS

$[\pi, 2\pi]$ is a valid branch for $\cos^{-1}$ but not for $\sin^{-1}$.

### TRAPS

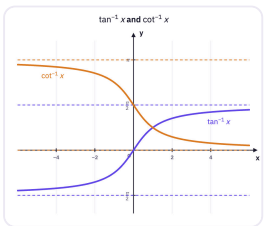
**✕ Giving $\cos^{-1}$ the range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$** $\cos x$ is not one-one there; $\cos^{-1}$ lives in $[0, \pi]$.

**✕ Giving $\cot^{-1}$ the range of $\tan^{-1}$** $\cot^{-1} x \in (0, \pi)$, so it is never negative.

**✕ Thinking any interval of length $\pi$ is a branch** It must be one-one and cover all of $[-1, 1]$: $[0, \pi]$ fails for sine.

## 2 Graphs of Inverse Trig Functions and Principal Values

- Graph of $f^{-1}$: reflect the principal branch in $y = x$.
- $\sin^{-1}$, $\tan^{-1}$ increase; $\cos^{-1}$, $\cot^{-1}$ decrease.
- Asymptotes: $\tan^{-1}$ at $y = \pm\frac{\pi}{2}$, $\cot^{-1}$ at $y = 0, \pi$.




### STANDARD RESULTS

$\operatorname{cosec}^{-1}(-2) = -\frac{\pi}{6}$, $\sec^{-1}(-\sqrt{2}) = \frac{3\pi}{4}$, $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \frac{2\pi}{3}$.

### TRAPS

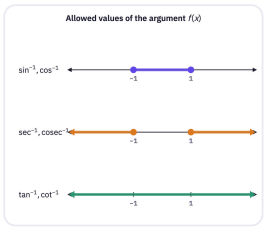
**✕ Writing $\cot^{-1}(-\sqrt{3}) = -\frac{\pi}{6}$** $\cot^{-1}$ is never negative: $\pi - \frac{\pi}{6} = \frac{5\pi}{6}$.

**✕ Writing $\sec^{-1}(-2) = -\frac{\pi}{3}$** $\sec^{-1}$ lives in $[0, \pi]$: $\pi - \frac{\pi}{3} = \frac{2\pi}{3}$.

**✕ Saying $\sec^{-1} x$ reaches $\frac{\pi}{2}$** That would need $\sec y = \infty$; the graph only approaches it.

## 3 Domain, Range and Inequalities of Inverse Trig Expressions

- Domain: bound the argument, keep the inner domain, intersect for sums.
- Inequality: apply the trig function; flip the sign for $\cos^{-1}$ and $\cot^{-1}$.
- Range: inner range → cut to the outer domain → carry through the monotonic function.




### STANDARD RESULTS

$\tan^{-1}(x^2 - 2x + 2) \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right)$; $(\tan^{-1} x)^2 - 3\tan^{-1} x + 2 \geq 0 \Leftrightarrow x \leq \tan 1$.

### TRAPS

**✕ Using $-1 \leq f(x) \leq 1$ for $\sec^{-1} f(x)$** It is the opposite region: $|f(x)| \geq 1$.

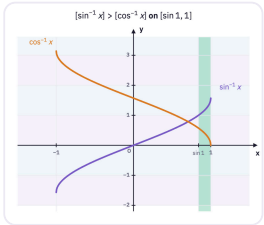
**✕ Forgetting the inner function's domain** A log still needs $x > 0$; a root still needs a non-negative radicand.

**✕ Not flipping the sign for $\cos^{-1}$ or $\cot^{-1}$** They are decreasing: $\cos^{-1} x < a \Rightarrow x > \cos a$.

**✕ Carrying all of $\mathbb{R}$ through $\sin^{-1}$** Cut the inner range to $[-1, 1]$ first.

## 4 Extreme-Value and Greatest Integer Equations in Inverse Trig

- A sum at its largest (smallest) possible value puts every term at its maximum (minimum).
- An argument that is never below 1 allows $\sin^{-1}$ or $\cos^{-1}$ at one point only.
- Greatest integer: list the possible integers, split, convert with monotonicity, compare numerically.



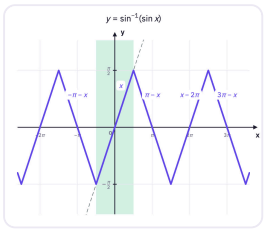
STANDARD RESULTS

$[\sin^{-1} x] > [\cos^{-1} x] \Leftrightarrow x \in [\sin 1, 1]$; $[\tan^{-1} x] = -1 \Leftrightarrow x \in [-\tan 1, 0)$.

TRAPS

- ✕ **Writing** $-2 \le \sin^{-1} x < -1$ **for** $[\sin^{-1} x] = -2$ $\sin^{-1} x \ge -\frac{\pi}{2}$, so the case is $-\frac{\pi}{2} \le \sin^{-1} x < -1$.
- ✕ **Pinning the sum but not each term** Only when the sum equals the largest possible total is every term forced to its maximum.
- ✕ **Comparing** sin 1 **and** cos 1 **by guesswork** Use numbers: $\cos 1 \approx 0.54$, $\sin 1 \approx 0.84$.

## 5 $\sin^{-1}(\sin x)$ and $\cos^{-1}(\cos x)$: Zig-Zag and Tent Graphs



- $f(f^{-1}(x)) = x$ on the domain of $f^{-1}$.
- $f^{-1}(f(x)) = x$ only on the principal branch.
- $\sin^{-1}(\sin x)$ is a zig-zag of period $2\pi$; $\cos^{-1}(\cos x)$ is an even tent of period $2\pi$.

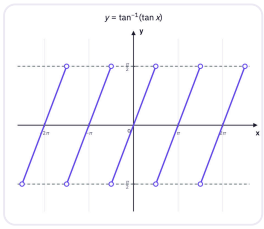
STANDARD RESULTS

$\sin^{-1}(\sin 10) = 3\pi - 10$, $\cos^{-1}(\cos 12) = 4\pi - 12$.

TRAPS

- ✕ **Writing** $\sin^{-1}(\sin x) = x$ **for every** $x$ Only on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$; elsewhere use the zig-zag.
- ✕ **Reading** sin 3 **as** sin 3° Without a degree sign the angle is in radians: 3 rad ≈ 172°.
- ✕ **Writing** $\sin(\sin^{-1} 2) = 2$ $\sin^{-1} 2$ does not exist, so the expression is undefined.

## 6 $\tan^{-1}(\tan x)$, $\cot^{-1}(\cot x)$, $\sec^{-1}(\sec x)$ and Solving with Graphs



- $\tan^{-1}(\tan x)$ and $\cot^{-1}(\cot x)$: period $\pi$, hollow ends.
- $\sec^{-1}(\sec x)$, $\operatorname{cosec}^{-1}(\operatorname{cosec} x)$ : tent and zig-zag with holes.
- Solve by drawing both sides; the right side must lie in the range.

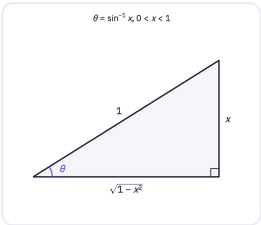
STANDARD RESULTS

$\sin^{-1}(\sin x) + \cos^{-1}(\cos x) = \pi$ on $\left[\frac{\pi}{2}, \pi\right]$; $\cos^{-1}(\cos x) = \frac{x}{3}$ has 3 solutions.

TRAPS

- ✕ **Giving** $\cot^{-1}(\cot x)$ **a negative value** Its range is $(0, \pi)$.
- ✕ **Forgetting the holes at odd multiples of** $\frac{\pi}{2}$ $\tan x$ and $\sec x$ do not exist there.
- ✕ **Counting crossings where the right side is out of range** The right side must lie in the left side's range.

## 7 The Reference Triangle: Converting Inverse Trig Functions



- Reference triangle: read every ratio of $\theta = f^{-1}(x)$, valid for $x > 0$.
- For $x < 0$ use $f^{-1}(-x)$: e.g. $\cos^{-1} x = \pi - \sin^{-1} \sqrt{1 - x^2}$.
- Ratios of $2\theta$, $\frac{\theta}{2}$: double- and half-angle formulas, sign from the quadrant.

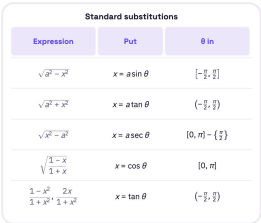
STANDARD RESULTS

$\sin\left(2 \tan^{-1} \frac{1}{2}\right) = \frac{4}{5}$; $\cos\left(\frac{1}{2} \cos^{-1}\left(-\frac{1}{9}\right)\right) = \frac{2}{3}$.

TRAPS

- ✕ **Using a triangle form for negative** $x$ Sides are positive; for $x < 0$ correct with $f^{-1}(-x)$.
- ✕ **Dropping the sign of a half-angle root** Find the quadrant of $\frac{\theta}{2}$ from the principal range of $\theta$.

## 8 Simplifying Inverse Trig Expressions by Substitution




- Choose the substitution from the root.
- Convert the $x$-interval into a $\theta$-interval before removing the inverse.
- Shift out-of-range angles; test the roots of equations.

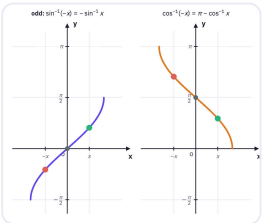
STANDARD RESULTS

$\tan^{-1} \frac{x}{1 + \sqrt{1 - x^2}} = \frac{1}{2} \sin^{-1} x$; $\tan^{-1} \sqrt{\frac{1 - x}{1 + x}} = \frac{1}{2} \cos^{-1} x$.

TRAPS

- ✕ **Writing** $\sqrt{\cos^2 \theta} = \cos \theta$ It is $|\cos \theta|$; the $\theta$-interval fixes the sign.
- ✕ **One formula for every** $x$ The same expression can need a shift by $\pi$ on another interval.
- ✕ **Not testing roots after substituting** Squaring or substituting can add roots that fail the original equation.

## 9 Inverse Trig of −x and 1/x



- Odd: $\sin^{-1}$, $\tan^{-1}$, $\operatorname{cosec}^{-1}$.
- $\pi - f^{-1}(x)$: $\cos^{-1}$, $\sec^{-1}$, $\cot^{-1}$.
- $\sin^{-1} \frac{1}{x} = \operatorname{cosec}^{-1} x$, $\cos^{-1} \frac{1}{x} = \sec^{-1} x$ for $|x| \ge 1$.

STANDARD RESULTS

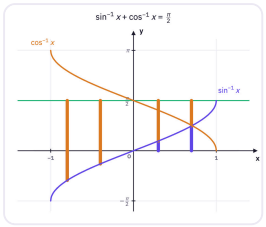
$\tan^{-1} x + \tan^{-1} \frac{1}{x} = \frac{\pi}{2}$ $(x > 0)$, $-\frac{\pi}{2}$ $(x < 0)$.

TRAPS

- ✕ **Writing** $\cot^{-1}(-x) = -\cot^{-1} x$ $\cot^{-1}$ belongs with $\cos^{-1}$: $\cot^{-1}(-x) = \pi - \cot^{-1} x$.

✕ **Using** $\tan^{-1}\frac{1}{x} = \cot^{-1} x$ **for** $x < 0$ There it is $-\pi + \cot^{-1} x$.

## 10 $\sin^{-1}x + \cos^{-1}x = \pi/2$ and the Complementary Identities



- Three complementary identities, each on its own domain.
- Proof: $\sin\theta = \cos\left(\frac{\pi}{2} - \theta\right)$ with $\frac{\pi}{2} - \theta$ in the range of $\cos^{-1}$.
- Replace the partner by $\frac{\pi}{2}$ minus the other, then use monotonicity.

### STANDARD RESULTS

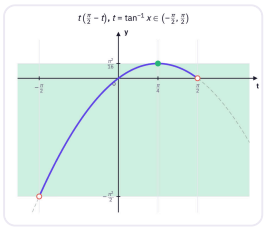
$\cos^{-1} x > \sin^{-1} x \Leftrightarrow x \in \left[-1, \frac{1}{\sqrt{2}}\right)$.

### TRAPS

✕ **Using an identity outside its domain** $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ needs $|x| \le 1$; $\sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2}$ needs $|x| \ge 1$.

✕ **Dropping the domain from an inequality's answer** $\cos^{-1} x > \sin^{-1} x$ gives $\left[-1, \frac{1}{\sqrt{2}}\right)$, not $\left(-\infty, \frac{1}{\sqrt{2}}\right)$.

## 11 Range and Extreme Values Using $\sin^{-1}x + \cos^{-1}x = \pi/2$



- Write the expression in $t = \sin^{-1} x$ (or $\tan^{-1} x$) with the partner $\frac{\pi}{2} - t$.
- Complete the square on $t$'s exact interval; check the vertex and both ends.
- Open ends for $\tan^{-1}$, $\cot^{-1}$; collapsing pairs leave a monotonic term.

### STANDARD RESULTS

$(\sin^{-1} x)^3 + (\cos^{-1} x)^3 \in \left[\frac{\pi^3}{32}, \frac{7\pi^3}{8}\right]$; $\sin^{-1} x \cdot \cos^{-1} x \in \left[-\frac{\pi^2}{2}, \frac{\pi^2}{16}\right]$.

### TRAPS

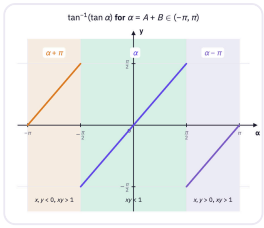
✕ **Closing the range at** $t = -\frac{\pi}{2}$ **for** $\tan^{-1}\tan^{-1} x$ never reaches $\pm\frac{\pi}{2}$.

✕ **Forgetting the domain cut to** $[-1, 1]$ Once $\sin^{-1} x$ or $\cos^{-1} x$ appears, the other terms only see $x \in [-1, 1]$.

✕ **Checking only the vertex** The other extreme is at an end of the interval.

## 12 $\tan^{-1}x + \tan^{-1}y$: The Addition Formula and Its Cases

- $A + B \in (-\pi, \pi)$; $\cos(A + B)$ has the sign of $1 - xy$.
- $xy < 1$: plain formula; $xy > 1$: add $\pi$ ($x, y > 0$) or subtract $\pi$ ($x, y < 0$).
- Difference: replace $y$ by $-y$; plain when $xy > -1$.



### STANDARD RESULTS

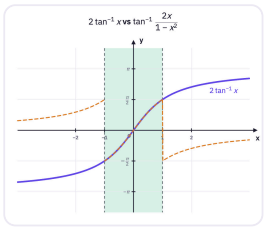
$\tan^{-1} 2 + \tan^{-1} 3 = \frac{3\pi}{4}$; $\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$.

### TRAPS

✕ **Using the** $xy < 1$ **formula when** $xy > 1$ Two positive angles would then add to a negative one.

✕ **Using** $xy < 1$ **as the condition for the difference** The difference formula needs $xy > -1$.

## 13 $2\tan^{-1}x$, $3\tan^{-1}x$ and $\tan^{-1}$ Equations



- $2\tan^{-1} x$: cut points $\pm 1$.
- $3\tan^{-1} x$: cut points $\pm\frac{1}{\sqrt{3}}$.
- Taking tan can add roots: substitute each back.

### STANDARD RESULTS

$3\tan^{-1}\frac{1}{2} = \tan^{-1}\frac{11}{2}$; $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4} \Rightarrow x = \frac{1}{6}$.

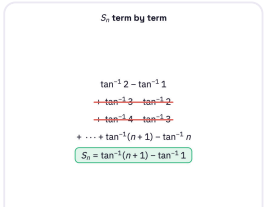
### TRAPS

✕ **Using the middle form of** $2\tan^{-1} x$ **for** $|x| > 1$ Outside $(-1, 1)$ add or subtract $\pi$.

✕ **Mixing the cut points of** $2\tan^{-1} x$ **and** $3\tan^{-1} x$ $\pm 1$ for the double, $\pm\frac{1}{\sqrt{3}}$ for the triple.

✕ **Not checking roots** Taking tan cannot tell $\theta$ from $\theta \pm \pi$.

## 14 Telescoping $\tan^{-1}$ Series



- Split the denominator as $1 + ab$ with $a - b =$ numerator.
- Find $S_n$; let $n \to \infty$ with $\tan^{-1}(\infty) = \frac{\pi}{2}$.
- Convert $\cot^{-1}$ terms to $\tan^{-1}$ first.

### STANDARD RESULTS

$\sum_{r=1}^{n} \tan^{-1}\frac{1}{1 + r + r^2} = \tan^{-1}\frac{n}{n+2}$; $\sum_{r=1}^{\infty} \cot^{-1}(r^2 + 5r + 7) = \tan^{-1}\frac{1}{3}$.

### TRAPS

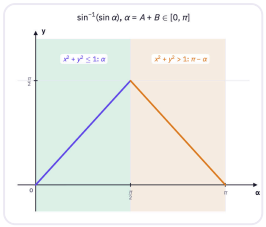
✕ **Writing the difference the wrong way round** Check that the $r = 1$ term matches the original term.

✕ **One survivor per end when the factors differ by 2** Then two terms survive at each end.

✕ **Leaving a** $\cot^{-1}$ **term unconverted** Use $\cot^{-1} y = \tan^{-1}\frac{1}{y}$ ($y > 0$) first.

## 15 Sum and Difference of $\sin^{-1}$ and $\cos^{-1}$

- $\sin^{-1}$ sum: formula if $x^2 + y^2 \leq 1$, else $\pi$ minus it ($x, y \geq 0$).
- $\cos^{-1}$ sum: no case split; difference: sign by $x \leq y$.
- Equations: isolate, take sin or cos, square, check signs.



### STANDARD RESULTS

$\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{12}{13} = \pi - \sin^{-1}\frac{56}{65}$; $\sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{3} \Rightarrow x = \frac{\sqrt{21}}{14}$.

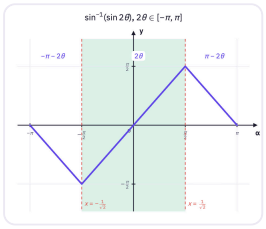
### TRAPS

**✕ Skipping the $x^2 + y^2 \leq 1$ check for a $\sin^{-1}$ sum**
When $x^2 + y^2 > 1$ the sum is $\pi$ minus the formula.

**✕ Keeping the wrong-sign root after squaring**
Squaring adds roots; check each in the original.

## 16 $2\sin^{-1}x$, $3\sin^{-1}x$, $2\cos^{-1}x$, $2\tan^{-1}x$: Where Each Case Comes From

- Substitute so the argument becomes $\sin k\theta$ or $\cos k\theta$.
- Read $\sin^{-1}(\sin\alpha)$ or $\cos^{-1}(\cos\alpha)$ on the possible $\alpha$ -interval.
- Translate the cut points back to $x$.



### STANDARD RESULTS

$\sin^{-1}\left(2x\sqrt{1-x^2}\right) = 2\sin^{-1} x \Leftrightarrow |x| \leq \frac{1}{\sqrt{2}}$.

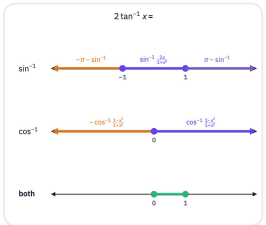
### TRAPS

**✕ Using the $|x| \leq 1$ form of $2\tan^{-1} x$ for $x > 1$**
For $x > 1$: $2\tan^{-1} x = \pi - \sin^{-1}\dfrac{2x}{1+x^2}$.

**✕ Mixing the cut points**
$2\sin^{-1} x$: $\pm\frac{1}{\sqrt{2}}$; $3\sin^{-1} x$: $\pm\frac{1}{2}$; $2\cos^{-1} x$: 0; $2\tan^{-1} x$: $\pm1$ or 0.

## 17 Multiple-Angle Formulas: Evaluating and Solving

- Pick the case from the value of $x$ first.
- Recognise $2\tan^{-1} x$ forms and use the range where each holds.
- Apply the $2\tan^{-1}$ form twice for $4\tan^{-1} x$.



### STANDARD RESULTS

For $x > 1$: $\sin^{-1}\frac{2x}{1+x^2} - \cos^{-1}\frac{1-x^2}{1+x^2} = \pi - 4\tan^{-1} x$.

### TRAPS

**✕ Replacing a form by $2\tan^{-1} x$ outside its valid range**
Check where $x$ is before replacing.

**✕ Dropping the case condition from the answer**
Conditions like $x > 1$ or $ab < 1$ belong in the final answer.

# Inverse Trigonometric Functions

$\sin^{-1}\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ $\cos^{-1}[0, \pi]$ $\tan^{-1}\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ $\cot^{-1}(0, \pi)$ $\pi \approx 3.14$ for locating sin 3, cos 10, tan 6

## 1 Why Inverse Trig Needs a Principal Value Branch

$\sin^{-1} : [-1, 1] \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\cos^{-1} : [-1, 1] \to [0, \pi]$

$\tan^{-1} : \mathbb{R} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\cot^{-1} : \mathbb{R} \to (0, \pi)$

$\sec^{-1} : |x| \geq 1 \to [0, \pi] - \left\{\frac{\pi}{2}\right\}$

$\operatorname{cosec}^{-1} : |x| \geq 1 \to \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

## 2 Graphs of Inverse Trig Functions and Principal Values

## 3 Domain, Range and Inequalities of Inverse Trig Expressions

## 4 Extreme-Value and Greatest Integer Equations in Inverse Trig

## 5 $\sin^{-1}(\sin x)$ and $\cos^{-1}(\cos x)$: Zig-Zag and Tent Graphs

$\sin^{-1}(\sin x) = x,\ |x| \leq \frac{\pi}{2}$

$\sin^{-1}(\sin x) = \pi - x,\ \frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$

$\sin^{-1}(\sin x) = x - 2\pi,\ \frac{3\pi}{2} \leq x \leq \frac{5\pi}{2}$

$\sin^{-1}(\sin x) = -\pi - x,\ -\frac{3\pi}{2} \leq x \leq -\frac{\pi}{2}$

$\cos^{-1}(\cos x) = x,\ 0 \leq x \leq \pi$

$\cos^{-1}(\cos x) = 2\pi - x,\ \pi \leq x \leq 2\pi$

$\cos^{-1}(\cos x) = x - 2\pi,\ 2\pi \leq x \leq 3\pi$

## 6 $\tan^{-1}(\tan x)$, $\cot^{-1}(\cot x)$, $\sec^{-1}(\sec x)$ and Solving with Graphs

## 7 The Reference Triangle: Converting Inverse Trig Functions

$\sin^{-1} x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\frac{x}{\sqrt{1-x^2}},\ 0 < x < 1$

$\tan^{-1} x = \sin^{-1}\frac{x}{\sqrt{1+x^2}} = \cos^{-1}\frac{1}{\sqrt{1+x^2}} = \cot^{-1}\frac{1}{x},\ x > 0$

## 8 Simplifying Inverse Trig Expressions by Substitution

## 9 Inverse Trig of −x and 1/x

$\sin^{-1}(-x) = -\sin^{-1} x$

$\tan^{-1}(-x) = -\tan^{-1} x$

$\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$

$\cos^{-1}(-x) = \pi - \cos^{-1} x$

$\sec^{-1}(-x) = \pi - \sec^{-1} x$

$\cot^{-1}(-x) = \pi - \cot^{-1} x$

$\tan^{-1}\frac{1}{x} = \cot^{-1} x,\ x > 0$

$\tan^{-1}\frac{1}{x} = -\pi + \cot^{-1} x,\ x < 0$

$\tan^{-1} x + \tan^{-1}\frac{1}{x} = \pm\frac{\pi}{2}\ (x \gtrless 0)$

## 10 $\sin^{-1}x + \cos^{-1}x = \pi/2$ and the Complementary Identities

$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2},\ |x| \leq 1$

$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2},\ x \in \mathbb{R}$

$\sec^{-1} x + \operatorname{cosec}^{-1} x = \frac{\pi}{2},\ |x| \geq 1$

## 11 Range and Extreme Values Using $\sin^{-1}x + \cos^{-1}x = \pi/2$

## 12 $\tan^{-1}x + \tan^{-1}y$: The Addition Formula and Its Cases

$\tan^{-1} x + \tan^{-1} y = \tan^{-1}\frac{x+y}{1-xy},\ xy < 1$

$= \pi + \tan^{-1}\frac{x+y}{1-xy},\ x, y > 0,\ xy > 1$

$= -\pi + \tan^{-1}\frac{x+y}{1-xy},\ x, y < 0,\ xy > 1$

$\tan^{-1} x - \tan^{-1} y = \tan^{-1}\frac{x-y}{1+xy},\ xy > -1$

$= \pi + \tan^{-1}\frac{x-y}{1+xy},\ x > 0 > y,\ xy < -1$

$= -\pi + \tan^{-1}\frac{x-y}{1+xy},\ x < 0 < y,\ xy < -1$

## 13 $2\tan^{-1}x$, $3\tan^{-1}x$ and $\tan^{-1}$ Equations

$2\tan^{-1} x = \tan^{-1}\frac{2x}{1-x^2}\ (|x| < 1);\ \pm\pi + \ldots\ (x \gtrless \pm 1)$

$3\tan^{-1} x = \tan^{-1}\frac{3x-x^3}{1-3x^2}\ \left(|x| < \frac{1}{\sqrt{3}}\right);\ \pm\pi + \ldots$

## 14 Telescoping $\tan^{-1}$ Series

$\tan^{-1}\frac{a-b}{1+ab} = \tan^{-1} a - \tan^{-1} b,\ ab > -1$

## 15 Sum and Difference of $\sin^{-1}$ and $\cos^{-1}$

$\sin^{-1} x + \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right),\ x^2 + y^2 \leq 1$

$= \pi - \sin^{-1}\left(x\sqrt{1-y^2} + y\sqrt{1-x^2}\right),\ x^2 + y^2 > 1$

$\cos^{-1} x + \cos^{-1} y = \cos^{-1}\left(xy - \sqrt{1-x^2}\sqrt{1-y^2}\right)$

$\cos^{-1} x - \cos^{-1} y = \pm\cos^{-1}\left(xy + \sqrt{1-x^2}\sqrt{1-y^2}\right)\ (x \leq y\ /\ x > y)$

## 16 $2\sin^{-1}x$, $3\sin^{-1}x$, $2\cos^{-1}x$, $2\tan^{-1}x$: Where Each Case Comes From

$2\sin^{-1} x = \sin^{-1}(2x\sqrt{1-x^2})\ \left(|x| \leq \frac{1}{\sqrt{2}}\right);\ \pm\pi - \ldots$

$3\sin^{-1} x = \sin^{-1}(3x - 4x^3)\ \left(|x| \leq \frac{1}{2}\right);\ \pm\pi - \ldots$

$2\cos^{-1} x = \cos^{-1}(2x^2 - 1)\ (x \geq 0);\ 2\pi - \cos^{-1}(2x^2 - 1)\ (x < 0)$

$2\tan^{-1} x = \sin^{-1}\frac{2x}{1+x^2}\ (|x| \leq 1);\ \pm\pi - \ldots$

$2\tan^{-1} x = \pm\cos^{-1}\frac{1-x^2}{1+x^2}\ (x \geq 0\ /\ x < 0)$

## 17 Multiple-Angle Formulas: Evaluating and Solving