Coulomb's Law and Electric Field

k 9×10^9 N m^{2}C^{-2} | ε_0 8.85×10^{-12} C^{2}N^{-1}m^{-2} | e 1.6×10^{-19} C | m_e 9.1×10^{-31} kg

1 Electric Charge: Properties and Methods of Charging

$i = dq/dt$

$1\ C = 1\ A\ s$

$\Delta m = n\, m_e$

$[Q] = [AT]$

$Q = \pm ne$

$e = 1.6 \times 10^{-19}$ C

2 Coulomb's Law and the Superposition Principle

$$F = k\frac{q_1 q_2}{r^2}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$$

$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 \varepsilon_r r^2}$$

$K = \varepsilon_r = \varepsilon/\varepsilon_0$

$F_{21} = (1/4\pi\varepsilon_0)\, q_1 q_2\, (r_2 - r_1)/|r_2 - r_1|^3$

$F_{12} = -F_{21}$

$F = F_1 + F_2 + \ldots + F_n$

$F = G\, m_1 m_2 / r^2$

$G = 6.67 \times 10^{-11}\ N\ m^2\ kg^{-2}$

3 Equilibrium of Point Charges

$q' = (q_1 + q_2)/2$

$$x = \frac{d\sqrt{q_1}}{\sqrt{q_1} + \sqrt{q_2}}$$

$$q_3 = -\frac{q_1 q_2}{(\sqrt{q_1} + \sqrt{q_2})^2}$$

$Q = -q/\sqrt{3}$ (triangle centroid)

$Q = -q(1 + 2\sqrt{2})/4$ (square centre)

$F_3 = -kQ[q_1(r - r_1)/|r - r_1|^3 + q_2(r - r_2)/|r - r_2|^3]$

4 Hanging Charged Balls and Mechanical Equilibrium

$$F_e = mg\tan\theta$$

$q = \sqrt{(16\pi\varepsilon_0\, l^2\, mg \tan\theta \sin^2\theta)}$

$x = (q^2 l/2\pi\varepsilon_0 mg)^{(1/3)}$

$$K = \frac{\rho}{\rho - \sigma}$$

$q = (4\pi\varepsilon_0 a^3 mg/3L)^{(1/2)}$

$Q = 4l\sqrt{(\pi\varepsilon_0 k(l - l_0))}$

$Q = mg(h^2 + l^2/2)^{(3/2)}/(4kh)$

$n = 4\pi\varepsilon_0 mg(h - R)^2/Q^2$

$\beta_{max} = \sin^{-1}(\mu \cot\alpha)$

5 Electric Field of Point Charges

$E = F/q_0$

$F = qE$

$[E] = [MLT^{-3}A^{-1}]$

$$E = \frac{kQ}{r^2}$$

$E = kQ(r_P - r_A)/|r_P - r_A|^3$

$E_y = 2kqy/(a^2 + y^2)^{(3/2)}$

$y_{max} = \pm a/\sqrt{2}$

$E = \Sigma\, kq_i(r - r_i)/|r - r_i|^3$

$r = (r_1\sqrt{q_2} + r_2\sqrt{q_1})/(\sqrt{q_1} + \sqrt{q_2})$

6 Charged Particles in a Uniform Electric Field

$q = mg \tan\theta/(A + B \tan\theta)$

$qE = mg$

$$T = 2\pi\sqrt{\frac{l}{g_{eff}}}$$

$g_{eff} = |g + qE/m|$

$a = qE/m$

$v = \sqrt{(2qEd/m)}$

$$y = \frac{qEl^2}{2mu^2}$$

$h = eEl^2/(2mv_0^2)$

$v = v_0\hat{i} + (eEl/mv_0)\hat{j}$

$H = 3eEl^2/(2mv_0^2)$

$qE\Delta t = \Delta p$

7 Continuous Charge and the Field of a Charged Ring

$dq = \lambda\, dl = \sigma\, dA = \rho\, dV$

$\lambda = Q/2\pi R$

$\sigma = Q/4\pi R^2$

$\rho = Q/(4/3)\pi R^3$

$$E = \frac{kQx}{(x^2 + a^2)^{3/2}}$$

$x_{max} = \pm a/\sqrt{2}$

$E_{max} = 2kQ/(3\sqrt{3}\, a^2)$

8 Line Charges, Rods and the Force Between Them

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$E_x = (\lambda/4\pi\varepsilon_0 r_0)(\sin\theta_1 + \sin\theta_2)$

$E_y = (\lambda/4\pi\varepsilon_0 r_0)(\cos\theta_1 - \cos\theta_2)$

$|E| = \sqrt{2}\lambda/(4\pi\varepsilon_0 r_0)$

$$E = \frac{kQ}{a(l + a)}$$

$F = (\lambda_1\lambda_2/2\pi\varepsilon_0)\ln(1 + l/r_0)$

$F = \lambda q/(4\pi\varepsilon_0 R)$

$F = \lambda_1\lambda_2/(2\varepsilon_0)$

9 Arcs, Disks, Sheets and Symmetry Shortcuts

$E = \lambda/(2\pi\varepsilon_0 R)$ (semicircle)

$E_x = E_y = \lambda/(4\pi\varepsilon_0 R)$ (quarter ring)

$E = \lambda \sin\theta/(2\pi\varepsilon_0 R)$ (symmetric arc)

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

$E = \sigma/\varepsilon_0$ (between oppositely charged sheets)

$E = \sigma x/(2\varepsilon_0\sqrt{(x^2 + R^2)})$ (sheet with hole)

$E = \sigma/(4\varepsilon_0)$ (hemisphere centre)

10 Electric Field Lines and Electrostatic Shielding

$\Omega = 2\pi(1 - \cos\alpha)$

$$\beta = 2\sin^{-1}\left[\sin\frac{\alpha}{2}\sqrt{\frac{q_1}{q_2}}\right]$$

11 Electric Dipole and Its Field

$p = q(2l)$

$E = (1/4\pi\varepsilon_0)\, 2pr/(r^2 - l^2)^2$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

$E = (1/4\pi\varepsilon_0)\, p/(r^2 + l^2)^{(3/2)}$

$E = -(1/4\pi\varepsilon_0)\, p/r^3$

$E = (p/4\pi\varepsilon_0 r^3)\sqrt{(1 + 3\cos^2\theta)}$

$\tan\varphi = \frac{1}{2}\tan\theta$

$F = 2kpQ/a^3$ (axial)

$F = kpQ/a^3$ (equatorial)

12 Dipole in Uniform and Non-uniform Fields

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$\tau = pE \sin\varphi$

$\tau_{max} = pE$

$T = 2\pi\sqrt{(I/(pE))}$

$$F = p\frac{dE}{dx}$$

$F = 6kp_1p_2/r^4$ (coaxial)

$F = 3kp_1p_2/r^4$ (side by side)

13 Small Oscillations of Charged Bodies

$$T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 ma^3}{qQ}}$$

$T = 2\pi\sqrt{(4\pi\varepsilon_0 mL^3/(Qq))}$

$T = 2\pi\sqrt{(mh^2/(2kq\lambda))}$

$\omega = \sqrt{(2g/y_0)}$

$T = 2\pi\sqrt{(m/(3E\lambda))}$

$T = (2\pi h/Q)\sqrt{(6\pi\varepsilon_0 Mh)}$

$f/f_0 = \sqrt{((3\eta - 2)/\eta)}$