# Coulomb's Law and Electric Field

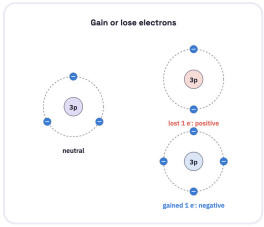
Last-minute revision: the key ideas, standard results and traps of every part. The formula sheet is the last page.

## If you only have five minutes

1. Charge: $Q = \pm ne$, conserved, additive; only repulsion proves charge.
2. Coulomb: $F = kq_1q_2/r^2$, $\div K$ in a medium; add forces as vectors.
3. Equilibrium in a line: nearer the smaller charge; hanging balls $F = mg\tan\theta$.
4. Field: $E = kQ/r^2$; ring $\frac{kQx}{(x^2 + a^2)^{3/2}}$; line $\frac{\lambda}{2\pi\varepsilon_0 r}$; sheet $\frac{\sigma}{2\varepsilon_0}$.
5. Dipole: axial $2kp/r^3$, equatorial $kp/r^3$; $\vec{\tau} = \vec{p} \times \vec{E}$; $F = p\,dE/dx$.

## 1 Electric Charge: Properties and Methods of Charging

- $Q = \pm ne$: quantized; conserved; additive with signs.
- Friction, conduction (share), induction (opposite charge, no contact).
- Only repulsion proves a body is charged.




### STANDARD RESULTS

2.7 g aluminium: $1.25 \times 10^5$ C of positive charge.

### TRAPS

✕ **Saying a charged body's mass is unchanged** It changes by $n\,m_e$: tiny, not zero.

✕ **Thinking conservation forbids creating charged particles** They are created in equal and opposite pairs.

✕ **Taking attraction as proof that a body is charged** A neutral body is attracted too; only repulsion proves charge.

✕ **Sharing charge equally between unequal spheres** Only identical spheres share equally.

## 2 Coulomb's Law and the Superposition Principle

- $F = kq_1q_2/r^2$, $k = 9 \times 10^9$; in a medium $\div K$.
- Vector form along $\vec{r}_2 - \vec{r}_1$; $\vec{F}_{12} = -\vec{F}_{21}$.
- Superposition: vector sum; a third charge doesn't change $F_{12}$.



### STANDARD RESULTS

2 µC, −3 µC at 30 cm: 0.6 N.

0.3 N ⊥ 0.4 N: 0.5 N.

### TRAPS

✕ **Using Coulomb's law exactly for large charged spheres close together** It holds for point charges.

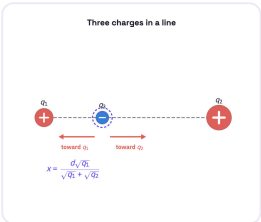
✕ **Dividing by $K$ when the medium fills only part of the space** Only a medium filling all the space gives $F/K$.

✕ **Using $\vec{r}_1 - \vec{r}_2$ for the force on $q_2$** Use $\vec{r}_2 - \vec{r}_1$ and $|\vec{r}_2 - \vec{r}_1|^3$.

✕ **Adding forces as numbers** Add them as vectors.

## 3 Equilibrium of Point Charges

- Like charges: balance between, nearer the smaller; unlike: outside.
- Whole system: $q_3 = -\frac{q_1q_2}{(\sqrt{q_1} + \sqrt{q_2})^2}$.
- Zero force can still be unstable.




### STANDARD RESULTS

+q, +9q at 40 cm: $-9q/16$ at 10 cm.

Square of 4 nC: −3.83 nC.

### TRAPS

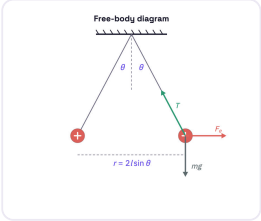
✕ **Placing the balance point between unlike charges** Both forces point the same way there.

✕ **Thinking the third charge's sign matters for its own balance** Only its position does.

✕ **Forgetting the diagonal charge on a square** It is $\sqrt{2}$ times farther: $F/2$.

## 4 Hanging Charged Balls and Mechanical Equilibrium

- $F = mg\tan\theta$, $r = 2l\sin\theta$.
- Small angles: $x^3 \propto q^2 l$.
- Liquid, same angle: $K = \rho/(\rho - \sigma)$.




### STANDARD RESULTS

10 g balls, 50 cm, 30°: 1.25 µC.

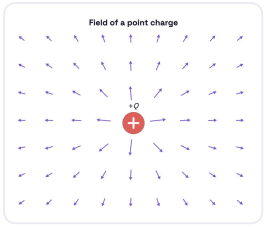
2.4 in 0.8 g/cm³: $K = 1.5$.

TRAPS

- ✕ **Taking the separation as $l \sin\theta$** It is $2l \sin\theta$.
- ✕ **Thinking the ball with more charge swings out further** Equal masses hang at equal angles.
- ✕ **Forgetting buoyancy in a liquid** Use the apparent weight $(\rho - \sigma)Vg$.

## 5 Electric Field of Point Charges

- $\vec{E} = \vec{F}/q_0$, N/C, $[MLT^{-3}A^{-1}]$.
- $E = kQ/r^2$, radial.
- Null point: nearer the smaller charge.




STANDARD RESULTS

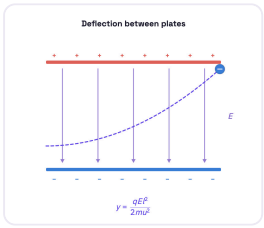
4 μC, 9 μC at 50 cm: E = 0 at 20 cm from 4 μC.

TRAPS

- ✕ **Pointing the field of a negative charge outward** It points toward the charge.
- ✕ **Using the position vector of P instead of $\vec{r}_P - \vec{r}_A$** The vector runs from the charge to the point.
- ✕ **Placing the null point between unlike charges** It lies outside, beyond the smaller charge.

## 6 Charged Particles in a Uniform Electric Field

- $a = qE/m$: projectile motion between plates.
- $y = qEl^2/2mu^2$; screen $l$ beyond: $H = 3h$.
- $\vec{g}_{\text{eff}} = \vec{g} + q\vec{E}/m$.




STANDARD RESULTS

Electron across 2 cm at 5000 N/C: $5.93 \times 10^6$ m/s.

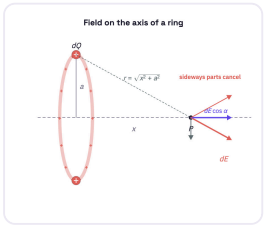
Pendulum with $qE/m = g$ down: 1.42 s.

TRAPS

- ✕ **Making a negative charge accelerate along $E$** It accelerates against $E$.
- ✕ **Assuming a charge moves along a curved field line** Its path is a parabola.
- ✕ **Adding $qE/m$ to $g$ when the force points up** Subtract it; if it exceeds $g$ the pendulum flips.

## 7 Continuous Charge and the Field of a Charged Ring

- $dq = \lambda\, dl = \sigma\, dA = \rho\, dV$.
- Ring: zero at the centre, max at $a/\sqrt{2}$.
- Far away: $kQ/x^2$.




STANDARD RESULTS

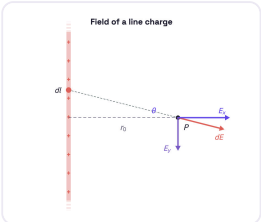
Ring 10 cm, 2 μC at 10 cm: $6.36 \times 10^5$ N/C.

TRAPS

- ✕ **Writing the field at a ring's centre as $kQ/a^2$** It is zero.
- ✕ **Integrating components that cancel by symmetry** Integrate only the axial part.

## 8 Line Charges, Rods and the Force Between Them

- Infinite line: $E = \lambda/2\pi\varepsilon_0 r$.
- Semi-infinite end: $\sqrt{2}\lambda/4\pi\varepsilon_0 r$ at 45°.
- Ring and thread: $F = \lambda q/4\pi\varepsilon_0 R$.




STANDARD RESULTS

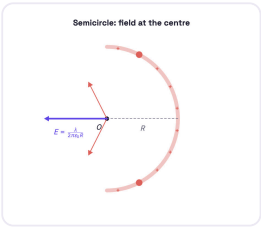
3 μC/m at 15 cm: $3.6 \times 10^5$ N/C.

TRAPS

- ✕ **Using $1/r^2$ for a line charge** It falls as $1/r$.
- ✕ **Swapping the sine and cosine sums** Sines give the perpendicular part, cosines the parallel part.
- ✕ **Using one field value along a whole second rod** Integrate $E\, dq$.

## 9 Arcs, Disks, Sheets and Symmetry Shortcuts

- Arc of half-angle θ: $\lambda \sin\theta/2\pi\varepsilon_0 R$.
- Disk → sheet: $\sigma/2\varepsilon_0$.
- Missing charge = opposite charge in the gap.




STANDARD RESULTS

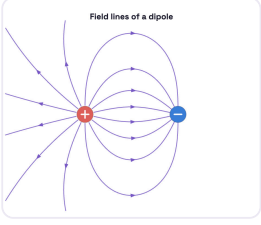
Semicircle 10 cm, 3.14 nC: 1800 N/C.

TRAPS

- ✕ **Using $\sigma/\varepsilon_0$ for a single sheet** One sheet gives $\sigma/2\varepsilon_0$.
- ✕ **Expecting an infinite sheet's field to fall with distance** It is uniform.
- ✕ **Measuring arc angles in degrees inside $R\theta$** Use radians.

## 10 Electric Field Lines and Electrostatic Shielding

- Tangent: direction; density: strength.
- Start on +, end on −; number ∝ charge.
- Earthed conductor shields; inside a hollow conductor E = 0.




STANDARD RESULTS

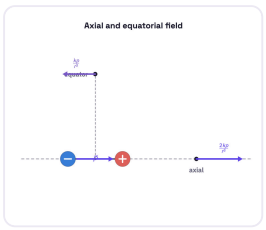
+4q, −q: neutral point d beyond −q.

TRAPS

- ✕ **Thinking a charge follows a curved field line** It does only if the line is straight.
- ✕ **Drawing two field lines crossing** E would have two directions there.
- ✕ **Thinking an unearthed neutral shell shields the outside** Its outer surface carries the inner charge.

## 11 Electric Dipole and Its Field

- $\vec{p}$ from − to +, C m.
- Axial = 2 × equatorial, opposite senses.
- $\tan\phi = \frac{1}{2}\tan\theta$.




STANDARD RESULTS

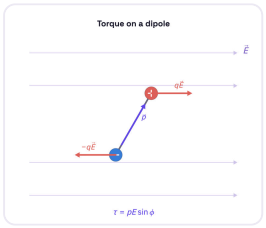
$p = 2 \times 10^{-9}$ at 30 cm: 1333 and 667 N/C.

TRAPS

- ✕ **Drawing $\vec{p}$ from + to −** It points from − to +.
- ✕ **Giving the equatorial field the direction of $\vec{p}$** It points opposite to $\vec{p}$.
- ✕ **Using $l$ for the separation when $l$ is half of it** $p = q \cdot 2l$.

## 12 Dipole in Uniform and Non-uniform Fields

- Uniform field: torque, no net force.
- Small oscillations: $T = 2\pi\sqrt{I/pE}$.
- Non-uniform: $F = p\,dE/dx$.




STANDARD RESULTS

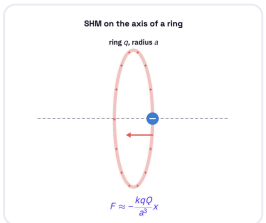
$p = 10^{-11}$ at 30° in $2 \times 10^{5}$: $10^{-6}$ N m.

TRAPS

- ✕ **Thinking a uniform field moves a dipole** The net force is zero; only a torque acts.
- ✕ **Using $\cos\phi$ in the torque** $\tau = pE\sin\phi$.
- ✕ **Sending a dipole along $\vec{E}$ toward the weaker field** It is pulled toward the stronger field.

## 13 Small Oscillations of Charged Bodies

- Small $x$: linear restoring force.
- Expand $(1 + x/h)^{-n} \approx 1 - nx/h$.
- Rigid bodies: $\tau = I\alpha$.




STANDARD RESULTS

Ring 10 cm, 10 µC; 10 g, −1 µC: 0.066 s.

TRAPS

- ✕ **Treating large displacements as SHM** Only small displacements give a linear force.
- ✕ **Expanding about the old equilibrium** Expand about the new one.
- ✕ **Using the wrong axis for the moment of inertia** Use the actual axis of rotation.

# Coulomb's Law and Electric Field

$k$ $9 \times 10^9$ N m$^2$C$^{-2}$ | $\varepsilon_0$ $8.85 \times 10^{-12}$ C$^2$N$^{-1}$m$^{-2}$ | $e$ $1.6 \times 10^{-19}$ C | $m_e$ $9.1 \times 10^{-31}$ kg

## 1 Electric Charge: Properties and Methods of Charging

$i = dq/dt$

$1\ C = 1\ A\ s$

$\Delta m = n\, m_e$

$[Q] = [AT]$

$Q = \pm ne$

$e = 1.6 \times 10^{-19}$ C

## 2 Coulomb's Law and the Superposition Principle

$$F = k\frac{q_1 q_2}{r^2}$$

$$k = \frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$$

$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2\ N^{-1}\ m^{-2}$

$$F = \frac{q_1 q_2}{4\pi\varepsilon_0 \varepsilon_r r^2}$$

$K = \varepsilon_r = \varepsilon/\varepsilon_0$

$F_{21} = (1/4\pi\varepsilon_0)\, q_1 q_2\, (r_2 - r_1)/|r_2 - r_1|^3$

$F_{12} = -F_{21}$

$F = F_1 + F_2 + \ldots + F_n$

$F = G\, m_1 m_2 / r^2$

$G = 6.67 \times 10^{-11}\ N\ m^2\ kg^{-2}$

## 3 Equilibrium of Point Charges

$q' = (q_1 + q_2)/2$

$$x = \frac{d\sqrt{q_1}}{\sqrt{q_1} + \sqrt{q_2}}$$

$$q_3 = -\frac{q_1 q_2}{(\sqrt{q_1} + \sqrt{q_2})^2}$$

$Q = -q/\sqrt{3}$ (triangle centroid)

$Q = -q(1 + 2\sqrt{2})/4$ (square centre)

$F_3 = -kQ[q_1(r - r_1)/|r - r_1|^3 + q_2(r - r_2)/|r - r_2|^3]$

## 4 Hanging Charged Balls and Mechanical Equilibrium

$$F_e = mg\tan\theta$$

$q = \sqrt{(16\pi\varepsilon_0\, l^2\, mg \tan\theta \sin^2\theta)}$

$x = (q^2 l/2\pi\varepsilon_0 mg)^{(1/3)}$

$$K = \frac{\rho}{\rho - \sigma}$$

$q = (4\pi\varepsilon_0 a^3 mg/3L)^{(1/2)}$

$Q = 4l\sqrt{(\pi\varepsilon_0 k(l - l_0))}$

$Q = mg(h^2 + l^2/2)^{(3/2)}/(4kh)$

$n = 4\pi\varepsilon_0 mg(h - R)^2/Q^2$

$\beta_{max} = \sin^{-1}(\mu \cot\alpha)$

## 5 Electric Field of Point Charges

$E = F/q_0$

$F = qE$

$[E] = [MLT^{-3}A^{-1}]$

$$E = \frac{kQ}{r^2}$$

$E = kQ(r_P - r_A)/|r_P - r_A|^3$

$E_y = 2kqy/(a^2 + y^2)^{(3/2)}$

$y_{max} = \pm a/\sqrt{2}$

$E = \Sigma\, kq_i(r - r_i)/|r - r_i|^3$

$r = (r_1\sqrt{q_2} + r_2\sqrt{q_1})/(\sqrt{q_1} + \sqrt{q_2})$

## 6 Charged Particles in a Uniform Electric Field

$q = mg \tan\theta/(A + B \tan\theta)$

$qE = mg$

$$T = 2\pi\sqrt{\frac{l}{g_{eff}}}$$

$g_{eff} = |g + qE/m|$

$a = qE/m$

$v = \sqrt{(2qEd/m)}$

$$y = \frac{qEl^2}{2mu^2}$$

$h = eEl^2/(2mv_0^2)$

$v = v_0\hat{i} + (eEl/mv_0)\hat{j}$

$H = 3eEl^2/(2mv_0^2)$

$qE\Delta t = \Delta p$

## 7 Continuous Charge and the Field of a Charged Ring

$dq = \lambda\, dl = \sigma\, dA = \rho\, dV$

$\lambda = Q/2\pi R$

$\sigma = Q/4\pi R^2$

$\rho = Q/(4/3)\pi R^3$

$$E = \frac{kQx}{(x^2 + a^2)^{3/2}}$$

$x_{max} = \pm a/\sqrt{2}$

$E_{max} = 2kQ/(3\sqrt{3}\, a^2)$

## 8 Line Charges, Rods and the Force Between Them

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$E_x = (\lambda/4\pi\varepsilon_0 r_0)(\sin\theta_1 + \sin\theta_2)$

$E_y = (\lambda/4\pi\varepsilon_0 r_0)(\cos\theta_1 - \cos\theta_2)$

$|E| = \sqrt{2}\lambda/(4\pi\varepsilon_0 r_0)$

$$E = \frac{kQ}{a(l + a)}$$

$F = (\lambda_1\lambda_2/2\pi\varepsilon_0)\ln(1 + l/r_0)$

$F = \lambda q/(4\pi\varepsilon_0 R)$

$F = \lambda_1\lambda_2/(2\varepsilon_0)$

## 9 Arcs, Disks, Sheets and Symmetry Shortcuts

$E = \lambda/(2\pi\varepsilon_0 R)$ (semicircle)

$E_x = E_y = \lambda/(4\pi\varepsilon_0 R)$ (quarter ring)

$E = \lambda \sin\theta/(2\pi\varepsilon_0 R)$ (symmetric arc)

$$E = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{\sqrt{x^2 + R^2}}\right]$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

$E = \sigma/\varepsilon_0$ (between oppositely charged sheets)

$E = \sigma x/(2\varepsilon_0\sqrt{(x^2 + R^2)})$ (sheet with hole)

$E = \sigma/(4\varepsilon_0)$ (hemisphere centre)

## 10 Electric Field Lines and Electrostatic Shielding

$\Omega = 2\pi(1 - \cos\alpha)$

$$\beta = 2\sin^{-1}\left[\sin\frac{\alpha}{2}\sqrt{\frac{q_1}{q_2}}\right]$$

## 11 Electric Dipole and Its Field

$p = q(2l)$

$E = (1/4\pi\varepsilon_0)\, 2pr/(r^2 - l^2)^2$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{2p}{r^3}$$

$E = (1/4\pi\varepsilon_0)\, p/(r^2 + l^2)^{(3/2)}$

$E = -(1/4\pi\varepsilon_0)\, p/r^3$

$E = (p/4\pi\varepsilon_0 r^3)\sqrt{(1 + 3\cos^2\theta)}$

$\tan\varphi = \frac{1}{2}\tan\theta$

$F = 2kpQ/a^3$ (axial)

$F = kpQ/a^3$ (equatorial)

## 12 Dipole in Uniform and Non-uniform Fields

$$\vec{\tau} = \vec{p} \times \vec{E}$$

$\tau = pE \sin\varphi$

$\tau_{max} = pE$

$T = 2\pi\sqrt{(I/(pE))}$

$$F = p\frac{dE}{dx}$$

$F = 6kp_1p_2/r^4$ (coaxial)

$F = 3kp_1p_2/r^4$ (side by side)

## 13 Small Oscillations of Charged Bodies

$$T = 2\pi\sqrt{\frac{4\pi\varepsilon_0 ma^3}{qQ}}$$

$T = 2\pi\sqrt{(4\pi\varepsilon_0 mL^3/(Qq))}$

$T = 2\pi\sqrt{(mh^2/(2kq\lambda))}$

$\omega = \sqrt{(2g/y_0)}$

$T = 2\pi\sqrt{(m/(3E\lambda))}$

$T = (2\pi h/Q)\sqrt{(6\pi\varepsilon_0 Mh)}$

$f/f_0 = \sqrt{((3\eta - 2)/\eta)}$