Electric Flux and Gauss's Law

ε_0 8.85×10^{-12} $C^2N^{-1}m^{-2}$ | k 9×10^9 $N\,m^2C^{-2}$

1 Electric Flux: Area Vector, Curved Surfaces and Integration

$$\Phi_E = EA$$

$$\Phi_E = EA\cos\theta = \vec{E}\cdot\vec{A}$$

$$d\Phi = \vec{E}\cdot d\vec{A}$$

$$\Phi_{\text{hemisphere}} = E\pi R^2$$

$$\Phi_{\text{cone, entering}} = ERh$$

$$\Phi_{\text{half-cylinder}} = 2ERl$$

$$\Phi = E(2RH\cos\theta + \pi R^2\sin\theta)$$

$$\Phi_{\text{tilted cube}} = EL^2(\cos\theta + \sin\theta)$$

$$\oint \vec{E}\cdot d\vec{A} = 0$$

$$\Phi = \int \vec{E}\cdot d\vec{A}$$

$$\Phi = \frac{\lambda R}{2\varepsilon_0}$$

$$\Phi = \frac{\lambda l}{\pi\varepsilon_0}\tan^{-1}\frac{b}{2d}$$

2 Gauss's Law: Enclosed Charge and Flux by Symmetry

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{\text{in}}}{\varepsilon_0}$$

$$\Phi = \frac{q_{\text{in}}}{K\varepsilon_0}$$

$$q_{\text{in}} = \varepsilon_0\,\Phi$$

$$q = \varepsilon_0\,b\,l^3$$

$$q = 4\pi\varepsilon_0 aR^3$$

$$\rho = \varepsilon_0\frac{dE}{dx}$$

$$\Phi_{\text{face}} = \frac{q}{6\varepsilon_0}\ \text{(charge at cube centre)}$$

$$\Phi_{\text{far face}} = \frac{q}{24\varepsilon_0}$$

$$\Phi = \frac{2\lambda\sqrt{R^2 - d^2}}{\varepsilon_0}$$

3 Conductors, Charged Shells and Charge in a Cavity

$$E = 0\ \text{(inside a conductor)}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

$$E = \frac{kq}{r^2}$$

$$E = 0\ \text{(inside)}$$

$$q_{\text{inner}} = -q$$

$$q_{\text{outer}} = Q + q$$

4 Coaxial and Concentric Conductors: Connecting and Earthing

$$\lambda_{\text{inner}} = -\lambda$$

$$\lambda_{\text{outer}} = \lambda_{\text{cyl}} + \lambda$$

$$E = \frac{\lambda + \lambda_{\text{cyl}}}{2\pi\varepsilon_0 r}$$

$$q_{\text{outer}}\ \text{(earthed)} = 0$$

5 Gauss's Law for a Line, a Solid Sphere and a Cylinder

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$E = \frac{2k\lambda}{r}$$

$$E = \frac{kQr}{R^3} = \frac{\rho r}{3\varepsilon_0}$$

$$E = \frac{kQ}{r^2}$$

$$\vec{E} = \frac{\rho\vec{r}}{3\varepsilon_0}$$

$$Q(r) = \int_0^r \rho(r')\,4\pi r'^2\,dr'$$

$$E = \frac{\rho r}{2\varepsilon_0}$$

$$E = \frac{\rho R^2}{2\varepsilon_0 r}$$

6 Charged Sheets and Conducting Plates

$$E = \frac{\sigma}{2\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$

$$q = \frac{Q}{2} \mp \varepsilon_0 EA$$

$$\sigma = \varepsilon_0(E_2 - E_1)$$

7 Parallel Conducting Plates, Earthing and Thick Slabs

$$q_{\text{outer}} = \frac{q_1 + q_2 + \cdots + q_n}{2}$$

$$q_{\text{facing}} = \pm\frac{q_1 - q_2}{2}$$

$$E = \frac{q_1 - q_2}{2A\varepsilon_0}$$

$$q_{\text{outer}} = 0\ \text{(any plate earthed)}$$

$$q_{\text{to earth}} = \sum Q_i$$

$$E = \frac{\rho x}{\varepsilon_0}$$

$$E = \frac{\rho d}{2\varepsilon_0}$$

8 Electrostatic Pressure: Force on a Charged Surface

$$p = \frac{\sigma^2}{2\varepsilon_0}$$

$$p = \frac{\varepsilon_0 E^2}{2}$$

$$F = \frac{\sigma^2\pi R^2}{2\varepsilon_0}$$

$$F = \frac{Q^2}{32\pi\varepsilon_0 R^2}$$

$$q_0 \geq \frac{q}{2}$$

$$\frac{F}{A} = \frac{\varepsilon_0(E_2^2 - E_1^2)}{2}$$

$$P_{\text{in}} + \frac{\sigma^2}{2\varepsilon_0} = P_0 + \frac{4T}{r}$$

9 Solid Angle and the Flux of a Point Charge

$$\Omega = \frac{S}{r^2}$$

$$d\Omega = \frac{dS\cos\theta}{r^2}$$

$$\Omega_{\text{sphere}} = 4\pi$$

$$\Omega = 2\pi(1 - \cos\alpha)$$

$$\Omega_{\text{disc}} = 2\pi\left(1 - \frac{a}{\sqrt{a^2 + R^2}}\right)$$

$$\Phi = \frac{q\Omega}{4\pi\varepsilon_0}$$

$$\Phi = \frac{q}{2\varepsilon_0}\left(1 - \frac{a}{\sqrt{a^2 + R^2}}\right)$$

10 Cavities and Superposition: Uniform Field Inside a Cavity

$$\vec{E} = \frac{\rho\vec{a}}{3\varepsilon_0}$$

$$\vec{E} = \frac{\rho\vec{d}}{3\varepsilon_0}$$

$$\vec{E} = \frac{\rho\vec{b}}{2\varepsilon_0}$$

$$\vec{E} = \frac{\rho}{3\varepsilon_0}\left[\vec{r} - \left(\frac{b}{|\vec{r} - \vec{a}|}\right)^3(\vec{r} - \vec{a})\right]$$

$$\vec{E} = \frac{\rho}{3\varepsilon_0}\left[\left(\frac{R}{r}\right)^3\vec{r} - \left(\frac{b}{|\vec{r} - \vec{a}|}\right)^3(\vec{r} - \vec{a})\right]$$

$$\omega^2 = \frac{\rho q}{3m\varepsilon_0}$$

$$T = 2\pi\sqrt{\frac{3m\varepsilon_0}{\rho q}}$$