Electric Flux and Gauss's Law

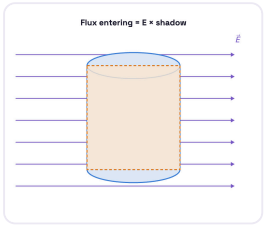
Last-minute revision: the key ideas, standard results and traps of every part. The formula sheet is the last page.

If you only have five minutes

1. Flux $\Phi = EA\cos\theta$ (θ from the normal); curved: $E\times$ shadow; closed, uniform field: 0.
2. Gauss: $\oint \vec{E}\cdot d\vec{A} = q_{in}/\varepsilon_0$; corner charge $q/8\varepsilon_0$, far face $q/24\varepsilon_0$.
3. Conductor: $E = 0$ inside, σ/ε_0 outside; cavity $-q$ inner, $Q + q$ outer.
4. Line $2k\lambda/r$; sphere kQr/R^3, kQ/r^2; sheet $\sigma/2\varepsilon_0$; plates: facing $\pm$, outer $\Sigma Q/2$.
5. Pressure $\sigma^2/2\varepsilon_0$; solid angle $2\pi(1-\cos\alpha)$; cavity $\rho\vec{a}/3\varepsilon_0$.

1 Electric Flux: Area Vector, Curved Surfaces and Integration

- $\Phi = EA\cos\theta$, θ from the normal; scalar, N m² C⁻¹.
- Curved surface: E × projected area; closed surface in a uniform field: zero.
- Varying field: integrate over strips where $E\cdot\hat{n}$ is constant.



STANDARD RESULTS

Tilted cylinder: $E(2RH\cos\theta + \pi R^2\sin\theta)$ enters.

TRAPS

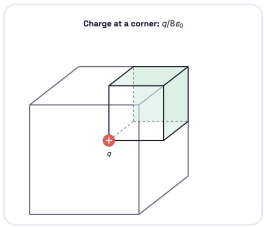
× Using the angle between E and the surface θ is measured from the normal; otherwise cos becomes sin.

× Taking $2\pi R^2E$ for a hemisphere in an axial field The curved part carries the same flux as the base, $E\pi R^2$.

× Saying zero net flux means zero field Lines can enter and leave; only the net is zero.

2 Gauss's Law: Enclosed Charge and Flux by Symmetry

- $\oint E\cdot dA = q_{in}/\varepsilon_0$ for any closed surface.
- Outside charges change E but not Φ; zero flux is not zero field.
- Find q face by face with outward normals; split flux by symmetry.



STANDARD RESULTS

Corner charge: $q/8\varepsilon_0$ for the cube, $q/24\varepsilon_0$ per far face.

TRAPS

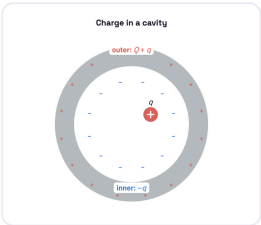
× Thinking E in Gauss's law is due only to enclosed charges E is the total field; only the flux is fixed by q_{in}.

× Saying Gauss's law needs symmetry to be true It always holds; symmetry is needed only to solve for E.

× Giving $q/6\varepsilon_0$ per face for an off-centre charge Only a charge at the cube's centre shares flux equally.

3 Conductors, Charged Shells and Charge in a Cavity

- Conductor: E = 0 inside, charge on the surface, $E = \sigma/\varepsilon_0$ outside.
- Sphere or thin shell: kq/r^2 outside, zero inside.
- Cavity charge q: $-q$ inner, $Q + q$ outer.



STANDARD RESULTS

Shell -3 µC with $+8$ µC inside: outer $+5$ µC, E(0.5 m) = 1.8×10^5 N/C.

TRAPS

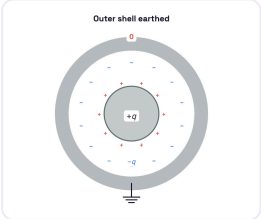
× Using $\sigma/2\varepsilon_0$ at a conductor's surface That is a thin sheet; a conductor gives σ/ε_0.

× Drawing E rising inside a conducting sphere That is the insulating sphere; a conductor has E = 0 inside.

× Putting the shell's own charge on its inner surface The inner surface carries only $-q$; the shell's charge goes outside.

4 Coaxial and Concentric Conductors: Connecting and Earthing

- Inner surface = −(enclosed charge); outer = the rest.
- Joined by a wire: all charge on the outermost surface.
- Earthed shell: outer surface 0, inner unchanged.



STANDARD RESULTS

$+3$ µC sphere in $+4$ µC shell: 7 µC flows to earth.

TRAPS

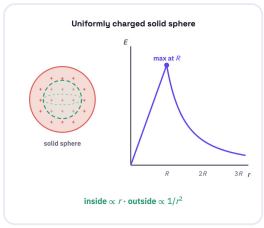
× Assuming an earthed conductor has zero charge It keeps whatever cancels the field of nearby charges.

× Clearing the inner surface when the outer shell is earthed Earthing empties only the outer surface.

✕ **Using only the tube's own charge for the field outside** Outside sees $\lambda + \lambda_{cyl}$.

5 Gauss's Law for a Line, a Solid Sphere and a Cylinder

- Pick the surface from the symmetry; E constant on each part.
- Line: $\lambda/2\pi\varepsilon_0 r = 2k\lambda/r$.
- Sphere: kQr/R^3 inside, kQ/r^2 outside; cylinder: $\rho r/2\varepsilon_0$, $\rho R^2/2\varepsilon_0 r$.



STANDARD RESULTS

4 μC sphere, R = 10 cm: E(5 cm) = 1.8×10^6, E(20 cm) = 9×10^5 N/C.

TRAPS

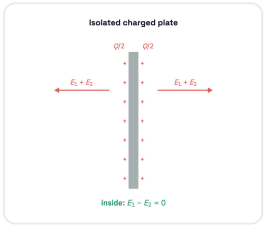
✕ **Using $1/r^2$ for a line charge** A line's field falls as $1/r$.

✕ **Using the conducting-sphere graph for an insulating sphere** Inside a uniform insulating sphere E rises linearly.

✕ **Writing $\rho r/3\varepsilon_0$ inside a cylinder** A cylinder gives $\rho r/2\varepsilon_0$.

6 Charged Sheets and Conducting Plates

- Sheet: $E = \sigma/2\varepsilon_0$, away on both sides, any distance.
- Lone plate: Q/2 per face, $Q/2A\varepsilon_0$ outside.
- In a field: $Q/2 \mp \varepsilon_0 EA$; $\sigma = \varepsilon_0(E_2 - E_1)$.



STANDARD RESULTS

6 μC plate, 0.2 m², 2×10^5 N/C: faces 2.65 and 3.35 μC.

TRAPS

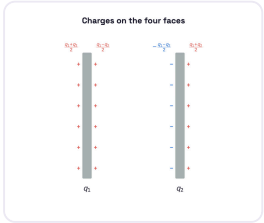
✕ **Swapping $\sigma/2\varepsilon_0$ and σ/ε_0** A sheet gives $\sigma/2\varepsilon_0$; a conductor's face σ/ε_0.

✕ **Putting all of a lone plate's charge on one face** It splits equally, Q/2 each.

✕ **Making a sheet's field fall with distance** An infinite sheet's field is uniform.

7 Parallel Conducting Plates, Earthing and Thick Slabs

- Facing faces equal and opposite; outer faces ΣQ/2.
- Earth any plate: outer faces 0; ΣQ flows to earth.
- Slab: $\rho x/\varepsilon_0$ inside, $\rho d/2\varepsilon_0$ outside.



STANDARD RESULTS

+5 and +1 μC plates: E between ≈ 1.13×10^6 N/C.

TRAPS

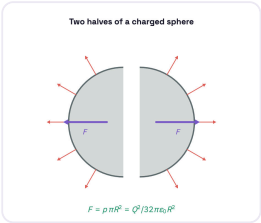
✕ **Making the outer faces unequal when one plate is neutral** Both outer faces always carry ΣQ/2.

✕ **Using $\sigma/2\varepsilon_0$ between plates** Two facing faces give σ/ε_0.

✕ **Setting the earthed plate's total charge to zero** Only its outer face goes to zero.

8 Electrostatic Pressure: Force on a Charged Surface

- A patch feels $\sigma/2\varepsilon_0$, not σ/ε_0.
- $p = \sigma^2/2\varepsilon_0 = \varepsilon_0 E^2/2$, always outward.
- Halves: $F = p\pi R^2 = Q^2/32\pi\varepsilon_0 R^2$.



STANDARD RESULTS

2 μC on R = 20 cm: p ≈ 0.89 N/m², F ≈ 0.11 N.

TRAPS

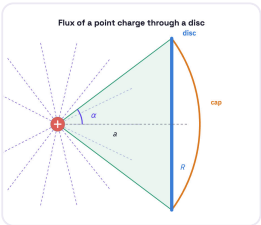
✕ **Using the full σ/ε_0 on the patch** Its own half doesn't push it: use $\sigma/2\varepsilon_0$.

✕ **Multiplying the pressure by $2\pi R^2$** The halves are pushed apart by $p \times \pi R^2$.

✕ **Thinking a negative shell is pulled inward by itself** The pressure is always outward.

9 Solid Angle and the Flux of a Point Charge

- $\Omega = S/r^2$; closed surface 4π, hemisphere 2π.
- Cone: $\Omega = 2\pi(1 - \cos\alpha)$, α the half-angle.
- $\Phi = q\Omega/4\pi\varepsilon_0$; disc fraction $(1 - \cos\alpha)/2$.



STANDARD RESULTS

17.7 nC, disc 6 cm at 8 cm: Φ = 200 N m² C⁻¹.

TRAPS

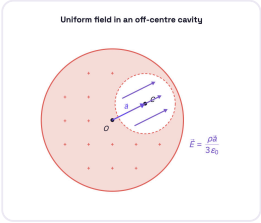
✕ **Taking $q/\varepsilon_0 \times$ (disc area)/$(4\pi a^2)$** The disc is flat; use the solid angle.

✕ **Using the full apex angle for α** α is the half-angle.

✕ **Forgetting cos θ for a tilted patch** $d\Omega = dS\cos\theta/r^2$.

10 Cavities and Superposition: Uniform Field Inside a Cavity

- Cavity = +ρ body + −ρ body.
- Sphere cavity $\rho a/3\varepsilon_0$, tunnel $\rho b/2\varepsilon_0$: uniform.
- Tunnel SHM: $T = 2\pi\sqrt{(3m\varepsilon_0/\rho q)}$.



STANDARD RESULTS

$\rho = 1\ \mu C/m^3$, $a = 6$ cm: $E \approx 2.26 \times 10^3$ N/C.

TRAPS

✕ **Expecting the cavity field to vary from point to point** It is $\rho a/3\varepsilon_0$ everywhere inside.

✕ **Using $3\varepsilon_0$ for a cylinder** A cylinder gives $\rho b/2\varepsilon_0$.

✕ **Using the given total charge as the full sphere's** Find ρ from the hollowed volume first.

Electric Flux and Gauss's Law

ε_0 8.85×10^{-12} $C^2N^{-1}m^{-2}$ | k 9×10^9 $N\,m^2C^{-2}$

1 Electric Flux: Area Vector, Curved Surfaces and Integration

$$\Phi_E = EA$$

$$\Phi_E = EA\cos\theta = \vec{E}\cdot\vec{A}$$

$$d\Phi = \vec{E}\cdot d\vec{A}$$

$$\Phi_{\text{hemisphere}} = E\pi R^2$$

$$\Phi_{\text{cone, entering}} = ERh$$

$$\Phi_{\text{half-cylinder}} = 2ERl$$

$$\Phi = E(2RH\cos\theta + \pi R^2\sin\theta)$$

$$\Phi_{\text{tilted cube}} = EL^2(\cos\theta + \sin\theta)$$

$$\oint \vec{E}\cdot d\vec{A} = 0$$

$$\Phi = \int \vec{E}\cdot d\vec{A}$$

$$\Phi = \frac{\lambda R}{2\varepsilon_0}$$

$$\Phi = \frac{\lambda l}{\pi\varepsilon_0}\tan^{-1}\frac{b}{2d}$$

2 Gauss's Law: Enclosed Charge and Flux by Symmetry

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{\text{in}}}{\varepsilon_0}$$

$$\Phi = \frac{q_{\text{in}}}{K\varepsilon_0}$$

$$q_{\text{in}} = \varepsilon_0\,\Phi$$

$$q = \varepsilon_0\,b\,l^3$$

$$q = 4\pi\varepsilon_0 aR^3$$

$$\rho = \varepsilon_0\frac{dE}{dx}$$

$$\Phi_{\text{face}} = \frac{q}{6\varepsilon_0}\ \text{(charge at cube centre)}$$

$$\Phi_{\text{far face}} = \frac{q}{24\varepsilon_0}$$

$$\Phi = \frac{2\lambda\sqrt{R^2 - d^2}}{\varepsilon_0}$$

3 Conductors, Charged Shells and Charge in a Cavity

$$E = 0\ \text{(inside a conductor)}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

$$E = \frac{kq}{r^2}$$

$$E = 0\ \text{(inside)}$$

$$q_{\text{inner}} = -q$$

$$q_{\text{outer}} = Q + q$$

4 Coaxial and Concentric Conductors: Connecting and Earthing

$$\lambda_{\text{inner}} = -\lambda$$

$$\lambda_{\text{outer}} = \lambda_{\text{cyl}} + \lambda$$

$$E = \frac{\lambda + \lambda_{\text{cyl}}}{2\pi\varepsilon_0 r}$$

$$q_{\text{outer}}\ \text{(earthed)} = 0$$

5 Gauss's Law for a Line, a Solid Sphere and a Cylinder

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$E = \frac{2k\lambda}{r}$$

$$E = \frac{kQr}{R^3} = \frac{\rho r}{3\varepsilon_0}$$

$$E = \frac{kQ}{r^2}$$

$$\vec{E} = \frac{\rho\vec{r}}{3\varepsilon_0}$$

$$Q(r) = \int_0^r \rho(r')\,4\pi r'^2\,dr'$$

$$E = \frac{\rho r}{2\varepsilon_0}$$

$$E = \frac{\rho R^2}{2\varepsilon_0 r}$$

6 Charged Sheets and Conducting Plates

$$E = \frac{\sigma}{2\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{2A\varepsilon_0}$$

$$q = \frac{Q}{2} \mp \varepsilon_0 EA$$

$$\sigma = \varepsilon_0(E_2 - E_1)$$

7 Parallel Conducting Plates, Earthing and Thick Slabs

$$q_{\text{outer}} = \frac{q_1 + q_2 + \cdots + q_n}{2}$$

$$q_{\text{facing}} = \pm\frac{q_1 - q_2}{2}$$

$$E = \frac{q_1 - q_2}{2A\varepsilon_0}$$

$$q_{\text{outer}} = 0\ \text{(any plate earthed)}$$

$$q_{\text{to earth}} = \sum Q_i$$

$$E = \frac{\rho x}{\varepsilon_0}$$

$$E = \frac{\rho d}{2\varepsilon_0}$$

8 Electrostatic Pressure: Force on a Charged Surface

$$p = \frac{\sigma^2}{2\varepsilon_0}$$

$$p = \frac{\varepsilon_0 E^2}{2}$$

$$F = \frac{\sigma^2\pi R^2}{2\varepsilon_0}$$

$$F = \frac{Q^2}{32\pi\varepsilon_0 R^2}$$

$$q_0 \geq \frac{q}{2}$$

$$\frac{F}{A} = \frac{\varepsilon_0(E_2^2 - E_1^2)}{2}$$

$$P_{\text{in}} + \frac{\sigma^2}{2\varepsilon_0} = P_0 + \frac{4T}{r}$$

9 Solid Angle and the Flux of a Point Charge

$$\Omega = \frac{S}{r^2}$$

$$d\Omega = \frac{dS\cos\theta}{r^2}$$

$$\Omega_{\text{sphere}} = 4\pi$$

$$\Omega = 2\pi(1 - \cos\alpha)$$

$$\Omega_{\text{disc}} = 2\pi\left(1 - \frac{a}{\sqrt{a^2 + R^2}}\right)$$

$$\Phi = \frac{q\Omega}{4\pi\varepsilon_0}$$

$$\Phi = \frac{q}{2\varepsilon_0}\left(1 - \frac{a}{\sqrt{a^2 + R^2}}\right)$$

10 Cavities and Superposition: Uniform Field Inside a Cavity

$$\vec{E} = \frac{\rho\vec{a}}{3\varepsilon_0}$$

$$\vec{E} = \frac{\rho\vec{d}}{3\varepsilon_0}$$

$$\vec{E} = \frac{\rho\vec{b}}{2\varepsilon_0}$$

$$\vec{E} = \frac{\rho}{3\varepsilon_0}\left[\vec{r} - \left(\frac{b}{|\vec{r} - \vec{a}|}\right)^3(\vec{r} - \vec{a})\right]$$

$$\vec{E} = \frac{\rho}{3\varepsilon_0}\left[\left(\frac{R}{r}\right)^3\vec{r} - \left(\frac{b}{|\vec{r} - \vec{a}|}\right)^3(\vec{r} - \vec{a})\right]$$

$$\omega^2 = \frac{\rho q}{3m\varepsilon_0}$$

$$T = 2\pi\sqrt{\frac{3m\varepsilon_0}{\rho q}}$$