Heat Transfer

σ 5.67×10^{-8} W m^{-2}K^{-4} · b 2.89×10^{-3} m K · k (W/m·K) Cu 385 · glass 0.8 · air 0.026 · T kelvin in T^4 and λT

1 Heat Conduction: Mechanism and Steady State

$$\frac{dT}{dx} = \text{constant}$$

- Steady state: heat in = heat out, T constant; uniform rod: linear fall.

2 Law of Heat Conduction and Thermal Conductivity

$$\frac{dQ}{dt} = \frac{kA(T_1 - T_2)}{l}$$

$$\frac{dQ}{dt} = -kA\frac{dT}{dx}$$

$$ms\frac{dT}{dt} = kA\left[\left(\frac{dT}{dx}\right)_A - \left(\frac{dT}{dx}\right)_B\right]$$

- k: copper 385, glass 0.8, air 0.026 W m^{-1} K^{-1}.

3 Thermal Resistance and Rods in Series

$$R_{th} = \frac{l}{kA}$$

$$T_1 - T_2 = \frac{dQ}{dt}R_{th}$$

$$\frac{dQ}{dt} = \frac{T_1 - T_2}{R_{th}}$$

$$T_x = \frac{k_1 l_2 T_1 + k_2 l_1 T_2}{k_1 l_2 + k_2 l_1}$$

$$R_{eq} = R_1 + R_2$$

$$k_{eq} = \frac{2k_1 k_2}{k_1 + k_2}$$

- Series: $R_{eq} = R_1 + R_2$; equal lengths $k_{eq} = \frac{2k_1 k_2}{k_1 + k_2}$.

4 Rods in Parallel and Networks of Rods

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$k_{eq} = \frac{k_1 A_1 + k_2 A_2}{A_1 + A_2}$$

$$\sum_i \frac{T_i - T_j}{R_{ij}} = 0$$

- Parallel: $\frac{1}{R_{eq}} = \sum \frac{1}{R_i}$; junction: $\sum \frac{T_i - T_j}{R_{ij}} = 0$.

5 Radial Conduction and the Growing Ice Layer

$$R = \int \frac{dx}{kA(x)}$$

$$R_{sphere} = \frac{b - a}{4\pi kab}$$

$$\frac{dQ}{dt} = \frac{4\pi kab(T_1 - T_2)}{b - a}$$

$$R_{cyl} = \frac{\ln(r_2/r_1)}{2\pi kL}$$

$$\frac{dQ}{dt} = \frac{k\pi r_1 r_2(T_1 - T_2)}{L}$$

$$\frac{kAT\,dt}{x} = A\rho L\,dx$$

$$t = \frac{\rho L}{2kT}(x_2^2 - x_1^2)$$

- Shells: sphere $\frac{b - a}{4\pi kab}$, cylinder $\frac{\ln(r_2/r_1)}{2\pi kL}$; ice $t \propto x_2^2 - x_1^2$.

6 Bodies Heated or Cooled Through a Rod

$$T = T_0 - (T_0 - T_1)e^{-KAt/msL}$$

$$T_1 - T_2 = (T_{10} - T_{20})e^{-\frac{kA}{l}\left(\frac{1}{m_1 s_1} + \frac{1}{m_2 s_2}\right)t}$$

$$T = T_0 + (T_1 - T_0)e^{-6ka^2(\gamma-1)t/nRx}$$

- Light rod: $T = T_0 - (T_0 - T_1)e^{-t/\tau}$, $\tau = \frac{msL}{KA}$.

7 Convection of Heat

$$\frac{dQ}{dt} = hA\Delta T$$

- Convection: $\frac{dQ}{dt} = hA\Delta T$ (h from experiment).

8 Thermal Radiation, Prevost's Theory and Kirchhoff's Law

$$E_I = E_R + E_A + E_T$$

$$a + r + t = 1$$

$$e = a$$

$$P = e\,P_{\text{black}}$$

- $a + r + t = 1$; Kirchhoff $e = a$; black body $e = a = 1$.

9 Stefan–Boltzmann Law and Net Radiation

$$E = \sigma T^4$$

$$P = \sigma A T^4$$

$$P = e\sigma A T^4$$

$$\frac{dQ}{dt} = e\sigma A(T^4 - T_s^4)$$

$$\frac{dT}{dt} = \frac{e\sigma A(T^4 - T_s^4)}{ms}$$

- $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}; rate of cooling $\propto A/(ms)$.

10 Radiation with Conduction, and the Solar Constant

$$\frac{k(T_A - T_B)}{l} = \sigma(T_B^4 - T_0^4)$$

$$E = \sigma(4\pi R_s^2)T_s^4$$

$$S = \frac{\sigma R_s^2 T_s^4}{d^2}$$

$$\frac{R_s}{d} = \frac{\theta}{2}$$

- Sun: $S = \sigma T_s^4(\theta/2)^2$, $T_s \approx 5800$ K.

11 Newton's Law of Cooling

$$\frac{dT}{dt} = -k(T - T_s)$$

$$k = \frac{4e\sigma A T_s^3}{ms}$$

$$T = T_s + (T_0 - T_s)e^{-kt}$$

$$\ln\frac{T - T_s}{T_0 - T_s} = -kt$$

$$\frac{T_1 - T_2}{t} = k\left[\frac{T_1 + T_2}{2} - T_s\right]$$

- Newton: $k = \frac{4e\sigma A T_s^3}{ms}$; average form with the mean T.

12 Comparing Cooling Bodies: Specific Heat and Steady Temperature

$$\frac{T_1 - T_2}{t} = \frac{K}{ms}\left[\frac{T_1 + T_2}{2} - T_s\right]$$

$$\frac{dQ}{dt} = \alpha(T - T_s)$$

- Same surface and range: cooling time $\propto ms$.

13 Black-Body Spectrum and Wien's Law

$$E = \int_0^\infty E_\lambda\,d\lambda = \sigma T^4$$

$$\lambda_m T = b$$

$$\lambda_{m1} T_1 = \lambda_{m2} T_2$$

$$E_\lambda(\text{max}) \propto T^5$$

$$\frac{E_1}{E_2} = \left(\frac{\lambda_{m2}}{\lambda_{m1}}\right)^5$$

- $b = 2.89 \times 10^{-3}$ m K; peak height $\propto T^5$, total $\propto T^4$.

14 Planck's Law and Kirchhoff's Law in Action

$$E_\lambda = \frac{2\pi ckT}{\lambda^4}$$

$$E_\lambda = \frac{2\pi hc^2}{\lambda^5(e^{hc/\lambda kT} - 1)}$$

$$\lambda_m = \frac{hc}{4.965\,kT}$$

$$\sigma = \frac{2\pi^5 k^4}{15c^2 h^3}$$

- Planck → Rayleigh (long λ), Wien, Stefan.