Heat Transfer

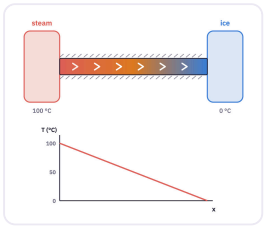
Last-minute revision: the key ideas, standard results and traps of every part. The formula sheet is the last page.

If you only have five minutes

1. Conduction: $\frac{dQ}{dt} = \frac{kA\Delta T}{l}$; $R = \frac{l}{kA}$; series adds R, parallel adds $1/R$.
2. Steady state: heat in = heat out at every point and junction; uniform rod, linear T.
3. Varying area: $R = \int \frac{dx}{kA}$; ice $t \propto x_2^2 - x_1^2$.
4. Radiation: $P = e\sigma AT^4$, net $e\sigma A(T^4 - T_s^4)$; Kirchhoff $e = a$.
5. Newton: $\frac{dT}{dt} = -k(T - T_s)$; Wien $\lambda_m T = b$; peak $\propto T^5$.

1 Heat Conduction: Mechanism and Steady State

- Conduction moves energy, not matter; metals conduct well through free electrons.
- Variable state: segments absorb heat; steady state: no segment does.
- Uniform rod in steady state: $\frac{dT}{dx}$ = constant (linear profile).



STANDARD RESULTS

20 cm rod, 100 → 0 °C: 70 °C at 6 cm from the hot end.

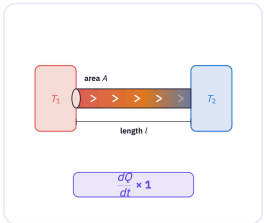
TRAPS

✕ Thinking molecules travel from the hot end to the cold end in conduction Only energy moves; each molecule stays in place.

✕ Calling steady state thermal equilibrium In steady state heat still flows; only the temperatures stop changing.

2 Law of Heat Conduction and Thermal Conductivity

- $\frac{dQ}{dt} = \frac{kA(T_1 - T_2)}{l} = -kA\frac{dT}{dx}$.
- k is a material property; unit W m^{-1} K^{-1}.
- Before steady state a section warms: $ms\frac{dT}{dt}$ = heat in − heat out.



STANDARD RESULTS

Boiler: heater ≈ 686 °C.

Stone slab: $k \approx 1.24$ W m^{-1} K^{-1}.

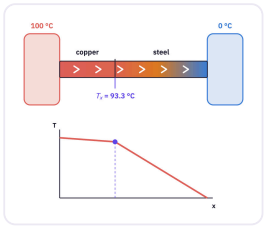
TRAPS

✕ Using the diameter instead of the radius, or cm instead of m, in $A = \pi r^2$ Use the radius, in metres.

✕ Using the wrong length l is measured along the heat flow (a slab's thickness, not its side).

3 Thermal Resistance and Rods in Series

- $R = \frac{l}{kA}$ and $\frac{dQ}{dt} = \frac{\Delta T}{R}$.
- Series: same current, resistances add.
- Bigger temperature drop across the poorer conductor.



STANDARD RESULTS

Copper–steel bar: $T_x \approx 93.3$ °C.

TRAPS

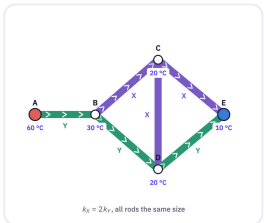
✕ Putting the junction at the average of the end temperatures Only true when the two resistances are equal.

✕ Adding conductivities for rods in series In series the resistances add; adding k is the parallel rule.

✕ Using $l/(kA)$ for a sphere or a cone When the area changes along the path, integrate $dR = \frac{dx}{kA}$.

4 Rods in Parallel and Networks of Rods

- Parallel: currents add, $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$.
- Two identical rods: 4× the heat side by side.
- Networks: heat in = heat out at each junction; equal temperatures exchange no heat.



STANDARD RESULTS

Box with plugs: 76 °C.

Network: 30, 20, 20 °C.

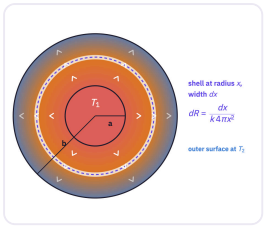
TRAPS

✕ Swapping the series and parallel rules Side by side the conductances add; end to end the resistances add.

✕ Missing a branch or mixing signs at a junction Count every branch and keep one direction convention.

5 Radial Conduction and the Growing Ice Layer

- Varying area: $R = \int \frac{dx}{kA(x)}$.
- Sphere $R = \frac{b-a}{4\pi kab}$; cylinder $R = \frac{\ln(r_2/r_1)}{2\pi kL}$.
- Ice: $t = \frac{\rho L}{2kT}(x_2^2 - x_1^2)$.



STANDARD RESULTS

Doubling ice thickness from zero takes 4× as long.

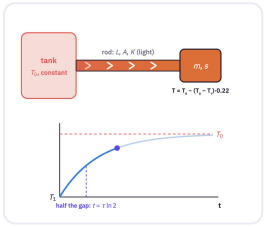
TRAPS

✕ **Using $l/(kA)$ with an average area for a sphere or cone** Integrate $dR = \frac{dx}{kA(x)}$.

✕ **Assuming equal thickness steps take equal times** $t \propto x_2^2 - x_1^2$: each layer takes longer.

6 Bodies Heated or Cooled Through a Rod

- A light rod is in steady state at every instant.
- $T = T_0 - (T_0 - T_1)e^{-KAt/msL}$.
- Time constant = heat capacity × thermal resistance.



STANDARD RESULTS

Doubling the rod length doubles every time.

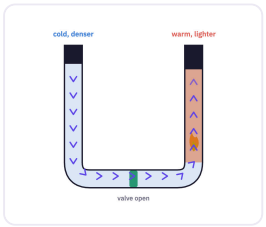
TRAPS

✕ **Using the quasi-steady formula for a heavy rod** It needs a rod of negligible heat capacity.

✕ **Integrating with the wrong sign** The temperature must approach T_0, not run away from it.

7 Convection of Heat

- Convection: heat carried by a moving fluid; solids can't convect.
- Natural (density-driven) or forced (fan, pump).
- $\frac{dQ}{dt} = hA\Delta T$; fins add area.



STANDARD RESULTS

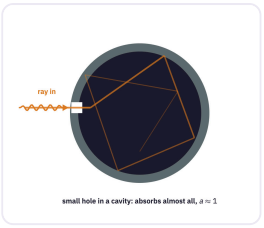
0.5 m² plate, ΔT = 40 K, h = 10: 200 W.

TRAPS

✕ **Saying convection can occur in solids** Convection needs a fluid; solids only conduct.

8 Thermal Radiation, Prevost's Theory and Kirchhoff's Law

- Every body above 0 K radiates and absorbs; net flow hot → cold.
- $a + r + t = 1$; e = P / P(black body).
- Kirchhoff: $e = a$; black body $e = a = 1$ (cavity with a hole).



STANDARD RESULTS

Reflects 30%, transmits 10%: $e = 0.6$.

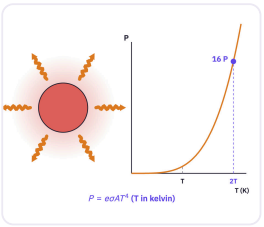
TRAPS

✕ **Thinking bodies in equilibrium stop emitting** They emit and absorb equally.

✕ **Calling a good reflector a good emitter** A good reflector is a poor absorber and so a poor emitter.

9 Stefan–Boltzmann Law and Net Radiation

- $P = e\sigma AT^4$, T in K.
- Net: $e\sigma A(T^4 - T_s^4)$; cooling rate divides by ms.
- Steady temperature: power in = net radiated.



STANDARD RESULTS

Sphere at 600 K in 300 K box: 0.206 °C/s.

Ball held at 1000 K: 21.2 W.

TRAPS

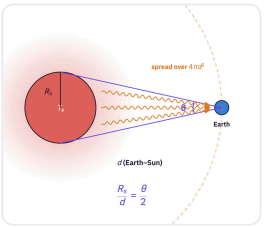
✕ **Writing $\sigma A(T - T_s)^4$** It is $\sigma A(T^4 - T_s^4)$.

✕ **Using °C in T^4, or forgetting e** Use kelvin, and include the emissivity of a non-black body.

✕ **Confusing rate of heat loss (∝ A) with rate of cooling (∝ A/ms)** Read which one the question asks for.

10 Radiation with Conduction, and the Solar Constant

- Steady state: heat conducted to a surface = net heat radiated from it.
- $S = \frac{\sigma R_s^2 T_s^4}{d^2} = \sigma T_s^4(\theta/2)^2$.
- $S \propto T^4$ and $S \propto \theta^2$.



STANDARD RESULTS

Rod into vacuum chamber: $k \approx 1.71$ W m⁻¹ °C⁻¹.

Sun: $T_s \approx 5810$ K.

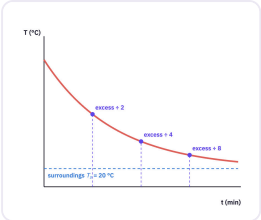
TRAPS

✕ **Using θ instead of θ/2 for R_s/d, or leaving θ in arc-minutes** $R_s/d = \theta/2$, in radians.

11 Newton's Law of Cooling

- Small differences: $\frac{dT}{dt} = -k(T - T_s)$.
- $T = T_s + (T_0 - T_s)e^{-kt}$: excess halves in equal times.

- Average form: $\frac{T_1 - T_2}{t} = k\left[\frac{T_1 + T_2}{2} - T_s\right]$.



STANDARD RESULTS

40 → 36 → 32 °C at 16 °C: 6.11 min.

Water 50 → 45 → 40 °C: $T_s \approx 34.2$ °C.

TRAPS

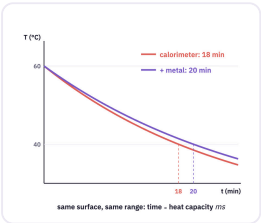
✕ **Using Newton's law when the body is far hotter than its surroundings** Use Stefan's law there.

✕ **Mixing up the cooling constant k with the thermal conductivity k** Same letter, different quantity.

✕ **Using the starting temperature instead of the mean in the average form** Use $\frac{T_1 + T_2}{2}$.

12 Comparing Cooling Bodies: Specific Heat and Steady Temperature

- $\frac{T_1 - T_2}{t} = \frac{K}{ms}[\bar{T} - T_s]$: time $\propto ms$.
- Method of cooling gives specific heats.
- Steady temperature: heater power = loss $\propto T - T_s$.



STANDARD RESULTS

Calorimeter + metal: $s \approx 0.185$ cal g^{-1} °C^{-1}.

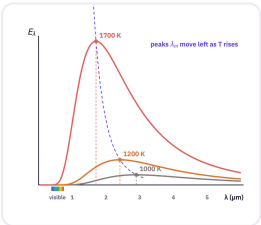
Heated ball: 20 W at 50 °C, 6.67 W at 30 °C.

TRAPS

✕ **Comparing cooling times of bodies with different surfaces** The method needs the same surface, surroundings and temperature range.

13 Black-Body Spectrum and Wien's Law

- Area under $E_\lambda = \sigma T^4$.
- Wien: $\lambda_m T = b$.
- Fifth power law: $E_\lambda(\text{max}) \propto T^5$.



STANDARD RESULTS

Sun from its peak: ≈ 5455 K.

100 W filament: $\lambda_m \approx 9972$ Å.

TRAPS

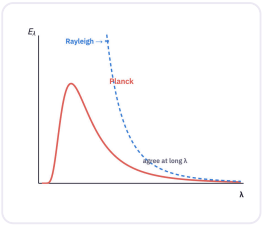
✕ **Using °C in $\lambda_m T = b$, or thinking λ_m grows with T** Use kelvin; λ_m falls as T rises.

✕ **Thinking a hotter body emits less at long wavelengths** Its curve lies above the cooler one at every wavelength.

✕ **Using T^4 for the peak spectral emittance** Peak $\propto T^5$; total $\propto T^4$.

14 Planck's Law and Kirchhoff's Law in Action

- Rayleigh fits only long λ; Planck fits all.
- $\frac{dE_\lambda}{d\lambda} = 0$ gives Wien; integrating gives Stefan.
- Kirchhoff: good absorbers are good emitters (Fraunhofer lines, deserts, glowing glass).



STANDARD RESULTS

Red glass heated in the dark glows green.

TRAPS

✕ **Quoting Rayleigh's formula as valid at all wavelengths** Only Planck's law fits the whole curve.

Heat Transfer

σ 5.67×10^{-8} W m^{-2}K^{-4} · b 2.89×10^{-3} m K · k (W/m·K) Cu 385 · glass 0.8 · air 0.026 · T kelvin in T^4 and λT

1 Heat Conduction: Mechanism and Steady State

$$\frac{dT}{dx} = \text{constant}$$

- Steady state: heat in = heat out, T constant; uniform rod: linear fall.

2 Law of Heat Conduction and Thermal Conductivity

$$\frac{dQ}{dt} = \frac{kA(T_1 - T_2)}{l}$$

$$\frac{dQ}{dt} = -kA\frac{dT}{dx}$$

$$ms\frac{dT}{dt} = kA\left[\left(\frac{dT}{dx}\right)_A - \left(\frac{dT}{dx}\right)_B\right]$$

- k: copper 385, glass 0.8, air 0.026 W m^{-1} K^{-1}.

3 Thermal Resistance and Rods in Series

$$R_{th} = \frac{l}{kA}$$

$$T_1 - T_2 = \frac{dQ}{dt}R_{th}$$

$$\frac{dQ}{dt} = \frac{T_1 - T_2}{R_{th}}$$

$$T_x = \frac{k_1 l_2 T_1 + k_2 l_1 T_2}{k_1 l_2 + k_2 l_1}$$

$$R_{eq} = R_1 + R_2$$

$$k_{eq} = \frac{2k_1 k_2}{k_1 + k_2}$$

- Series: $R_{eq} = R_1 + R_2$; equal lengths $k_{eq} = \frac{2k_1 k_2}{k_1 + k_2}$.

4 Rods in Parallel and Networks of Rods

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$k_{eq} = \frac{k_1 A_1 + k_2 A_2}{A_1 + A_2}$$

$$\sum_i \frac{T_i - T_j}{R_{ij}} = 0$$

- Parallel: $\frac{1}{R_{eq}} = \sum \frac{1}{R_i}$; junction: $\sum \frac{T_i - T_j}{R_{ij}} = 0$.

5 Radial Conduction and the Growing Ice Layer

$$R = \int \frac{dx}{kA(x)}$$

$$R_{sphere} = \frac{b - a}{4\pi kab}$$

$$\frac{dQ}{dt} = \frac{4\pi kab(T_1 - T_2)}{b - a}$$

$$R_{cyl} = \frac{\ln(r_2/r_1)}{2\pi kL}$$

$$\frac{dQ}{dt} = \frac{k\pi r_1 r_2(T_1 - T_2)}{L}$$

$$\frac{kAT\,dt}{x} = A\rho L\,dx$$

$$t = \frac{\rho L}{2kT}(x_2^2 - x_1^2)$$

- Shells: sphere $\frac{b - a}{4\pi kab}$, cylinder $\frac{\ln(r_2/r_1)}{2\pi kL}$; ice $t \propto x_2^2 - x_1^2$.

6 Bodies Heated or Cooled Through a Rod

$$T = T_0 - (T_0 - T_1)e^{-KAt/msL}$$

$$T_1 - T_2 = (T_{10} - T_{20})e^{-\frac{kA}{l}\left(\frac{1}{m_1 s_1} + \frac{1}{m_2 s_2}\right)t}$$

$$T = T_0 + (T_1 - T_0)e^{-6ka^2(\gamma-1)t/nRx}$$

- Light rod: $T = T_0 - (T_0 - T_1)e^{-t/\tau}$, $\tau = \frac{msL}{KA}$.

7 Convection of Heat

$$\frac{dQ}{dt} = hA\Delta T$$

- Convection: $\frac{dQ}{dt} = hA\Delta T$ (h from experiment).

8 Thermal Radiation, Prevost's Theory and Kirchhoff's Law

$$E_I = E_R + E_A + E_T$$

$$a + r + t = 1$$

$$e = a$$

$$P = e\,P_{\text{black}}$$

- $a + r + t = 1$; Kirchhoff $e = a$; black body $e = a = 1$.

9 Stefan–Boltzmann Law and Net Radiation

$$E = \sigma T^4$$

$$P = \sigma A T^4$$

$$P = e\sigma A T^4$$

$$\frac{dQ}{dt} = e\sigma A(T^4 - T_s^4)$$

$$\frac{dT}{dt} = \frac{e\sigma A(T^4 - T_s^4)}{ms}$$

- $\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}; rate of cooling $\propto A/(ms)$.

10 Radiation with Conduction, and the Solar Constant

$$\frac{k(T_A - T_B)}{l} = \sigma(T_B^4 - T_0^4)$$

$$E = \sigma(4\pi R_s^2)T_s^4$$

$$S = \frac{\sigma R_s^2 T_s^4}{d^2}$$

$$\frac{R_s}{d} = \frac{\theta}{2}$$

- Sun: $S = \sigma T_s^4(\theta/2)^2$, $T_s \approx 5800$ K.

11 Newton's Law of Cooling

$$\frac{dT}{dt} = -k(T - T_s)$$

$$k = \frac{4e\sigma A T_s^3}{ms}$$

$$T = T_s + (T_0 - T_s)e^{-kt}$$

$$\ln\frac{T - T_s}{T_0 - T_s} = -kt$$

$$\frac{T_1 - T_2}{t} = k\left[\frac{T_1 + T_2}{2} - T_s\right]$$

- Newton: $k = \frac{4e\sigma A T_s^3}{ms}$; average form with the mean T.

12 Comparing Cooling Bodies: Specific Heat and Steady Temperature

$$\frac{T_1 - T_2}{t} = \frac{K}{ms}\left[\frac{T_1 + T_2}{2} - T_s\right]$$

$$\frac{dQ}{dt} = \alpha(T - T_s)$$

- Same surface and range: cooling time $\propto ms$.

13 Black-Body Spectrum and Wien's Law

$$E = \int_0^\infty E_\lambda\,d\lambda = \sigma T^4$$

$$\lambda_m T = b$$

$$\lambda_{m1} T_1 = \lambda_{m2} T_2$$

$$E_\lambda(\text{max}) \propto T^5$$

$$\frac{E_1}{E_2} = \left(\frac{\lambda_{m2}}{\lambda_{m1}}\right)^5$$

- $b = 2.89 \times 10^{-3}$ m K; peak height $\propto T^5$, total $\propto T^4$.

14 Planck's Law and Kirchhoff's Law in Action

$$E_\lambda = \frac{2\pi ckT}{\lambda^4}$$

$$E_\lambda = \frac{2\pi hc^2}{\lambda^5(e^{hc/\lambda kT} - 1)}$$

$$\lambda_m = \frac{hc}{4.965\,kT}$$

$$\sigma = \frac{2\pi^5 k^4}{15c^2 h^3}$$

- Planck → Rayleigh (long λ), Wien, Stefan.