Oscillations and Simple Harmonic Motion

g 9.8 m/s² (or 10, π^2 where stated) | k_e $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9$ N m²C⁻² | R_e 6400 km

1 Periodic Motion, Oscillations and When Motion Is SHM

$v = \frac{1}{T}$

$a = -\omega^2 y$

$F = -ky$

$a = -\omega^2 y$

2 Equation of SHM, Phase, Velocity and Acceleration

$y = A\sin(\omega t + \alpha)$

$T = \frac{2\pi}{\omega}$

$v = \omega\sqrt{A^2 - y^2}$

$v_{max} = A\omega$

$a = -\omega^2 y$

$a_{max} = \omega^2 A$

3 Reading SHM Equations and Apparent Weight on an Oscillating Platform

$A\sin\omega t + B\cos\omega t = R\sin(\omega t + \phi)$

$R = \sqrt{A^2 + B^2}$

$\tan\phi = \frac{B}{A}$

$K_{max} = \frac{1}{2}m\omega^2 A^2$

$\bar{v} = \frac{2v_{max}}{\pi}$

4 Superposition of SHMs: Same and Different Frequencies

$R = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$\tan\theta = \frac{A_2\sin\phi}{A_1 + A_2\cos\phi}$

5 Angular SHM and the Three-Step Method to Find the Time Period

$\dot{\theta} = \omega\sqrt{\theta_0^2 - \theta^2}$

$\ddot{\theta} = -\omega^2\theta$

$\tau = -I\omega^2\theta$

$F_R = -kx \Rightarrow \omega = \sqrt{k/m}$

$\tau_R = -k\theta \Rightarrow \omega = \sqrt{k/I}$

$T = \frac{2\pi}{\omega}$

6 Simple Pendulum and Spring-Block Pendulum

$T = 2\pi\sqrt{\frac{l}{g}}$

$T_{max} = mg(3 - 2\cos\theta_0)$

$T_{min} = mg\cos\theta_0$

$\omega = \sqrt{\frac{k}{m}}$

$T = 2\pi\sqrt{\frac{m}{k}}$

7 Compound and Torsional Pendulums, Equivalent Length and Minimum Period

$T = 2\pi\sqrt{\frac{I}{mgl}}$

$\omega = \sqrt{\frac{C}{I}}$

$T = 2\pi\sqrt{\frac{I}{C}}$

$l_{eq} = l_c + \frac{k^2}{l_c}$

$T = 2\pi\sqrt{\frac{l_c + k^2/l_c}{g}}$

8 Time Period in Physical Situations: Strings, Effective g and Liquids

$F_R = -\frac{4T}{l}x$

$T = 2\pi\sqrt{\frac{l}{g - qE/m}}$

$T = 2\pi\sqrt{\frac{l}{g_{eff}}}$

$g_{eff} = \sqrt{g^2 + a^2}$

$T = 2\pi\sqrt{\frac{m}{A\rho g}}$

$T = 2\pi\sqrt{\frac{L}{2g}}$

9 Springs in Series and Parallel, Cut Springs, Pulleys and Rods on Springs

$k_{\parallel} = k_1 + k_2$

$\frac{1}{k_s} = \frac{1}{k_1} + \frac{1}{k_2}$

$k \propto \frac{1}{l}$ (cut spring)

$\tau_R = -\left(2kl^2 + \frac{mgl}{2}\right)\theta$

$\omega = \sqrt{\frac{12kl + 3mg}{2ml}}$

10 Phase Difference and Time Between Two Positions in SHM

$\phi = (\omega_1 - \omega_2)t$

$\phi = \sin^{-1}\frac{y}{A}$ or $\pi - \sin^{-1}\frac{y}{A}$

$d_{max} = 2A\sin\frac{\Delta\phi}{2}$

$t = \frac{\Delta\theta}{\omega}$

11 Sudden Changes in SHM: Collisions, Added Mass and Elastic Walls

$A = \sqrt{l^2\alpha^2 + \frac{v^2 l}{g}}$

$\Delta\phi = \sin^{-1}\frac{l\beta}{A_2} + \sin^{-1}\frac{l\alpha}{A_1} - \pi$

$v' = \frac{mv}{m+m'}$

$A' = \sqrt{x^2 + \frac{v'^2}{\omega'^2}}$

$A' = A\sqrt{\frac{m}{m+m'}}$ (at the mean)

$T = \sqrt{\frac{m}{k}}\left[\pi + 2\sin^{-1}\frac{l}{A}\right]$

$T = 2\sqrt{\frac{l}{g}}\left[\frac{\pi}{2} + \sin^{-1}\frac{\alpha}{\beta}\right]$

12 Energy in SHM: Potential Well, Kinetic and Potential Energy

$F = -\frac{dU}{dx}$

$K = \frac{1}{2}m\omega^2(A^2 - x^2)$

$E = \frac{1}{2}m\omega^2 A^2 = \frac{1}{2}kA^2$

$K = U$ at $x = \frac{A}{\sqrt{2}}$

$E = \frac{1}{2}I\omega^2\theta_0^2$

13 Energy Method to Find the Frequency: Pulleys and Rolling Bodies

$\omega = \sqrt{\frac{k_{eff}}{M_{eff}}}$

$T = 2\pi\sqrt{\frac{m + I/r^2}{k}}$

$T = 2\pi\sqrt{\frac{M + I/R^2}{4k}}$

$T = \frac{\pi}{2}\sqrt{\frac{3M}{k}}$

14 SHM in a Given Potential and Mean Kinetic Energy

$T = 2\pi\sqrt{\frac{m}{U_0 C^2}}$

$T = 2\pi\sqrt{\frac{R(M + m + I/R^2)}{mg\cos\alpha}}$

$\langle K\rangle = \frac{1}{4}I\omega^2\beta^2$

$\langle K\rangle = \frac{1}{8}mgl\theta_0^2 + \frac{1}{12}ml^2\dot{\theta}_0^2$

15 Taylor's Method: ω From the Force or Potential Function

$\omega = \sqrt{\frac{f'(x_0)}{m}}$

$\omega = \sqrt{\frac{U''(x_0)}{m}}$

$T = 2\pi\sqrt{\frac{mr^3}{2k_e Qq}}$

$T = 2\pi\sqrt{\frac{m}{2k}}$

16 SHM About a Shifted Mean Position: Constant Forces and Lifts

$y_m = \frac{C}{\omega^2}$

$y = l + \frac{F}{k}(1 - \cos\omega t)$

$v_{max} = \frac{F}{\sqrt{mk}}$

$A = \frac{ma}{k}$ (lift)

$\omega = \sqrt{\frac{k}{m} + \frac{2k_e q_1 q_2}{my_0^3}}$

17 Two-Body Oscillations, the Bifilar Rod and the Tunnel Through the Earth

$\mu = \frac{m_1 m_2}{m_1 + m_2}$

$T = 2\pi\sqrt{\frac{\mu}{k}}$

$T = 2\pi\sqrt{\frac{l}{3g}}$

$g_{in} = \frac{GM_e x}{R_e^3}$

$T = 2\pi\sqrt{\frac{R_e^3}{GM_e}} = 2\pi\sqrt{\frac{R_e}{g}}$